Dynamic Hypercube Topology

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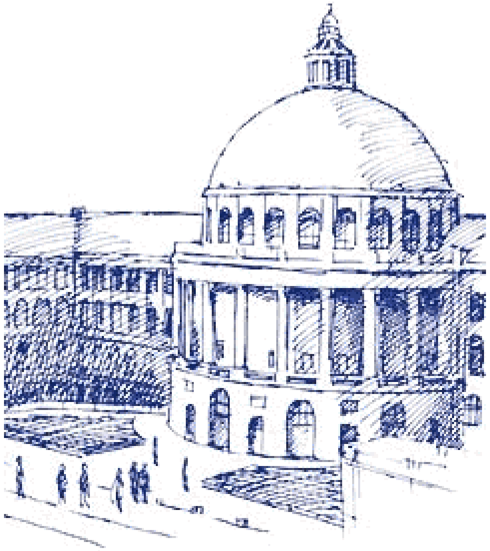

ETH
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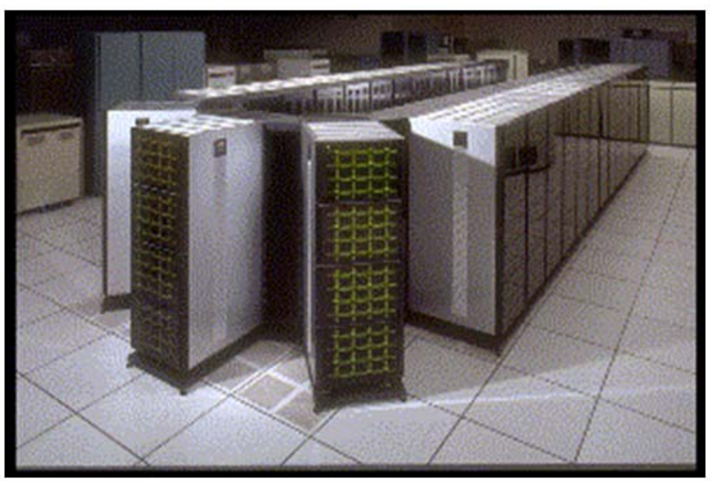
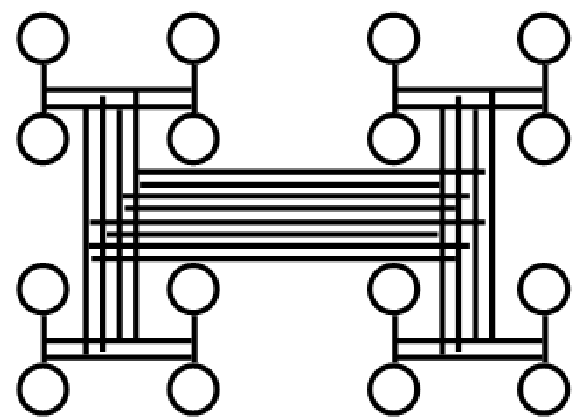
Static vs. Dynamic Networks (1)

- Network graph *G=(V,E)*
  - *V* = set of vertices ("nodes", machines, peers, …)
  - *E* = set of edges ("connections", wires, links, pointers, …)

- "Traditional", static networks
  - Fixed set of vertices, fixed set of edges
  - E.g., interconnection network of parallel computers



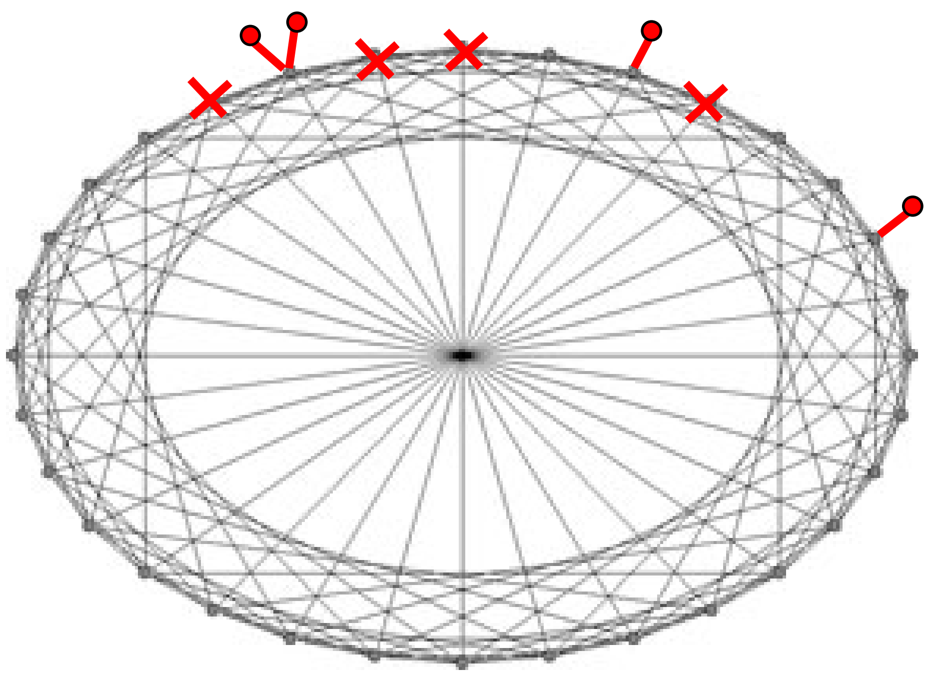
Parallel Computer



Fat Tree Topology

Static vs. Dynamic Networks (2)

- Dynamic networks
  - Set of nodes and/or set of edges is dynamic
  - Here: nodes may join and leave
  - E.g., peer-to-peer (P2P) systems (Napster, Gnutella, …)

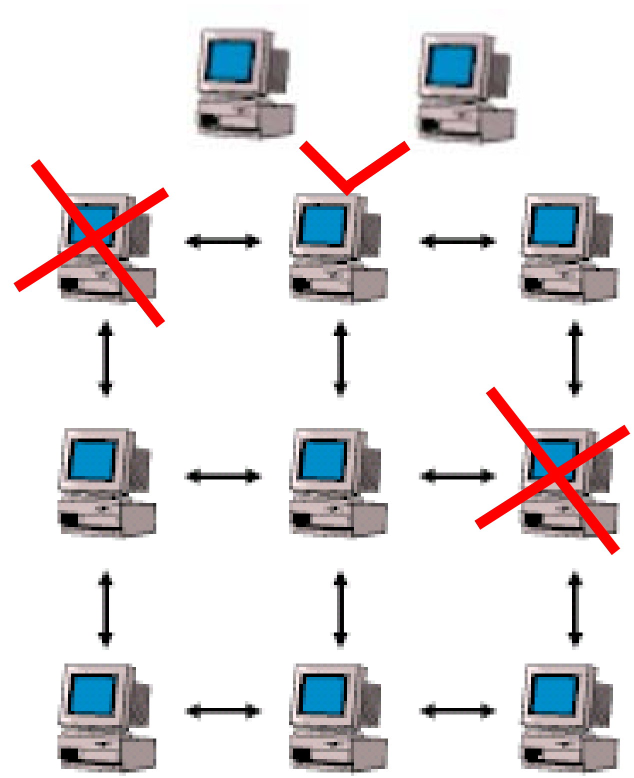


Dynamic Chord Topology

Dynamic Peer-to-Peer Systems

Peer-to-Peer Systems

- cooperation of many machines (to share files, CPU cycles, etc.)
- usually desktop computers under control of individual users
- user may turn machine on and off at any time
- => Churn



How to maintain desirable properties such as connectivity, network diameter, node degree, ...?

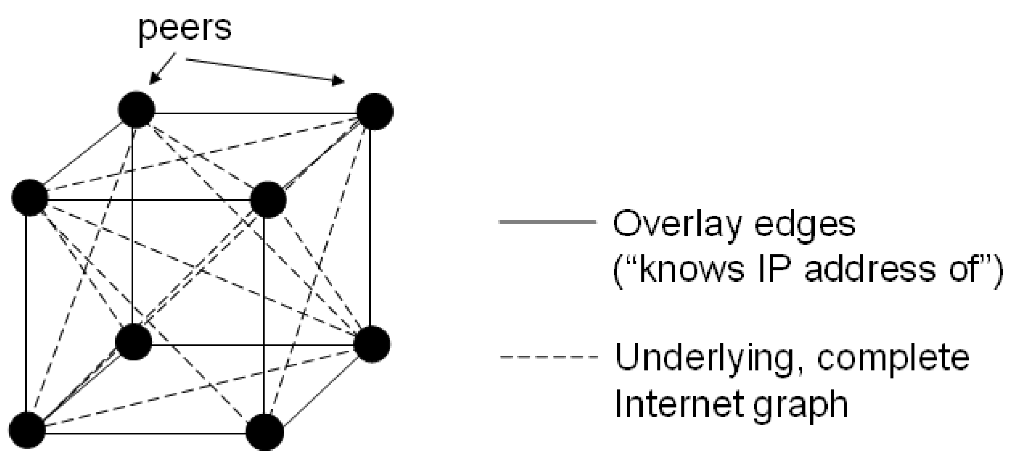
Talk Overview

- Model
- Ingredients: basic algorithms on hypercube graph
- Assembling the components
- Results for the hypercube
- Conclusion, generalization and open problems
- Discussion

Model (1): Network Model

- Typical P2P overlay network
  - Vertices $v \in V$: peers (dynamic: may join and leave)
  - Directed edges $(u,v) \in E$: u knows IP address of v (static)

- Assumption: Overlay network builds upon *complete* Internet graph
  - Sending a message over an overlay edge => routing in the underlying Internet graph

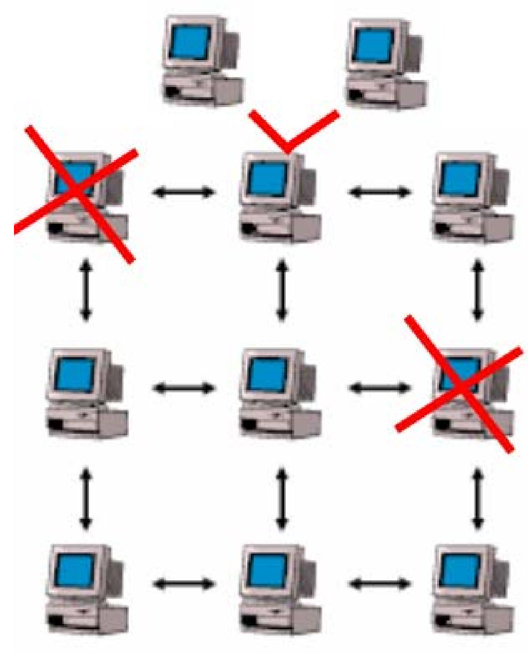


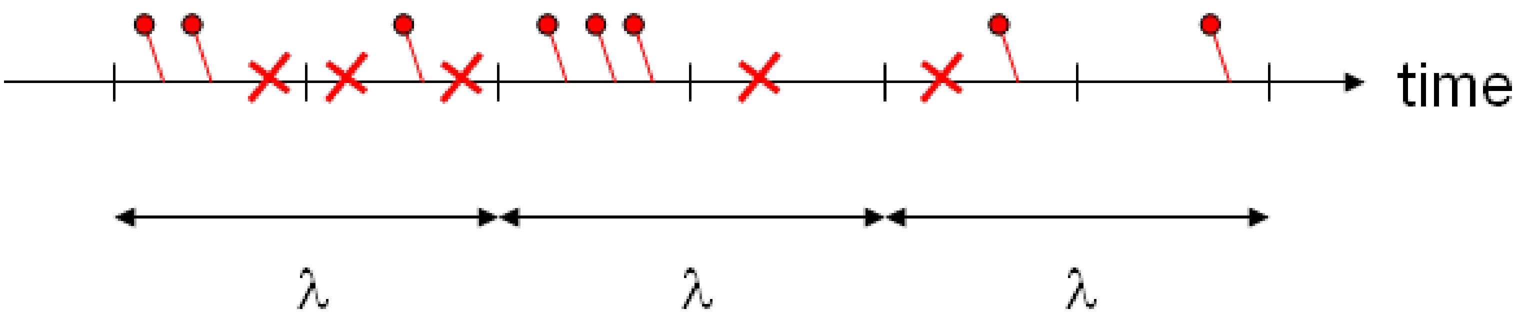
Model (2): Worst-Case (Adversarial) Dynamics

- Model worst-case faults with an adversary *ADV(J,L,λ)*

- *ADV(J,L,λ)* has complete visibility of the entire state of the system

- May add at most *J* and remove at most *L* peers in any time period of length λ



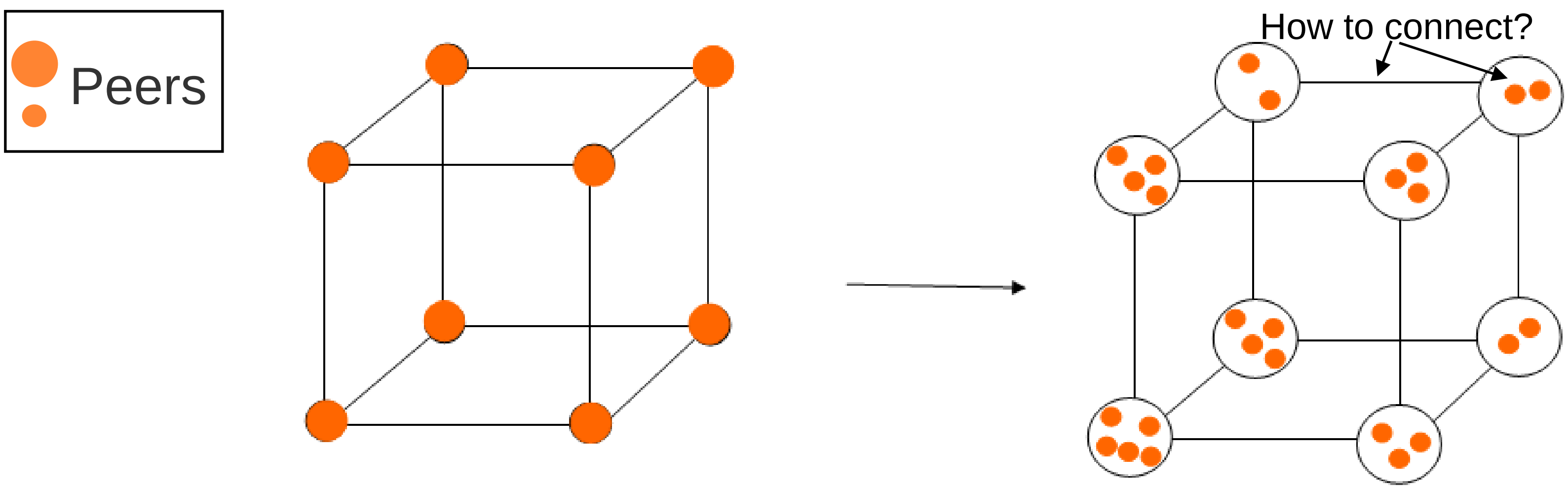


Model (3): Communication Rounds

- Our system is synchronous, i.e., our algorithms run in rounds
  - One round: receive messages, local computation, send messages
- However: Real distributed systems are asynchronous!
- But: Notion of time necessary to bound the adversary

Overview of Dynamic Hypercube System

- Idea: Arrange peers into a *simulated* hypercube where each node consists of *several* (logarithmically many) peers!
  - Gives a certain redundancy and thus time to react to changes.
  - But still guarantees diameter *D = O(log n)* and degree *Δ = O(log n)*, as in the normal hypercube (*n* = total number of peers)!



Normal Hypercube Topology

Simulated Hypercube Topology

Ingredients for Fault-Tolerant Hypercube System

Simulation: Node consists of several peers!

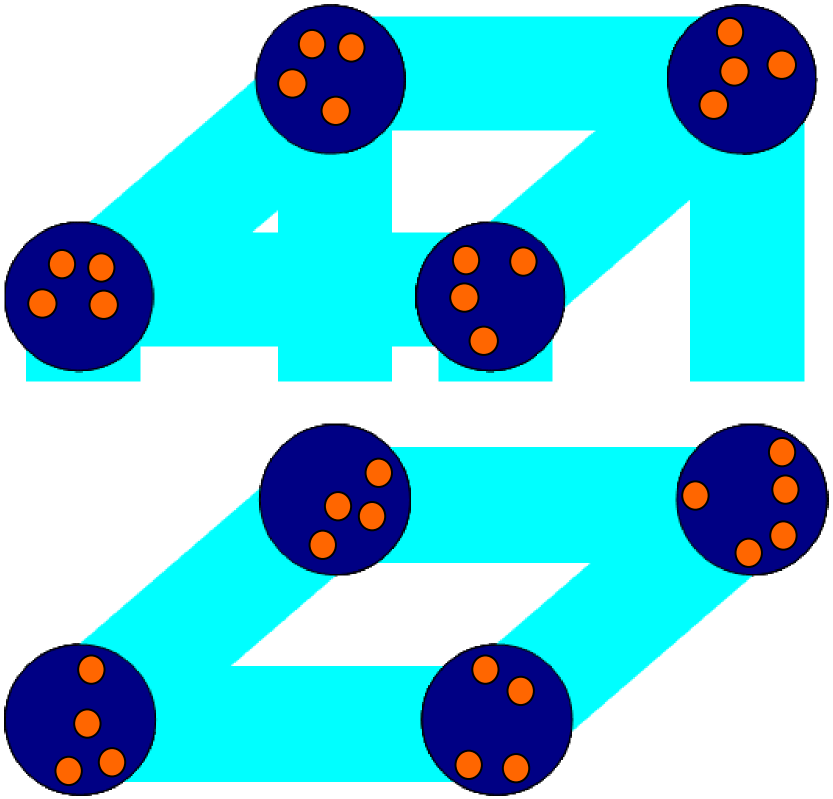
Basic components:

- Route peers to sparse areas

Token distribution algorithm!

- Adapt dimension

Information aggregation algorithm!



Components: Peer Distribution and Information Aggregation

Peer Distribution

Tackled next!

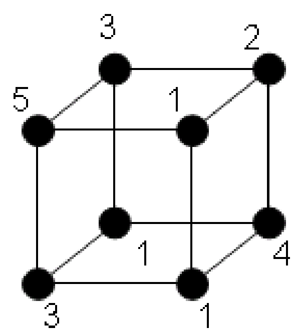
- Goal: Dis[illegible] peers evenly among all hypercube nodes in orde[illegible]ance biased adversarial churn
- [illegible]ally a token distribution problem

Counting the total number of peers (information aggregation)

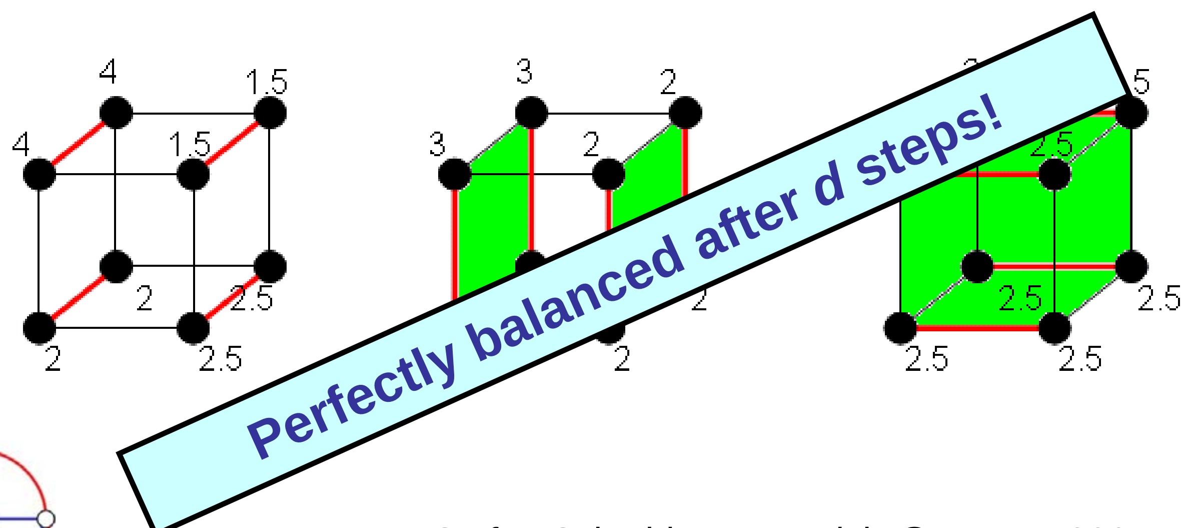
- Goal: Estimate the total number of peers in the system and adapt the dimension accordingly

Dynamic Token Distribution Algorithm (1)

Algorithm: Cycle over dimensions and balance!

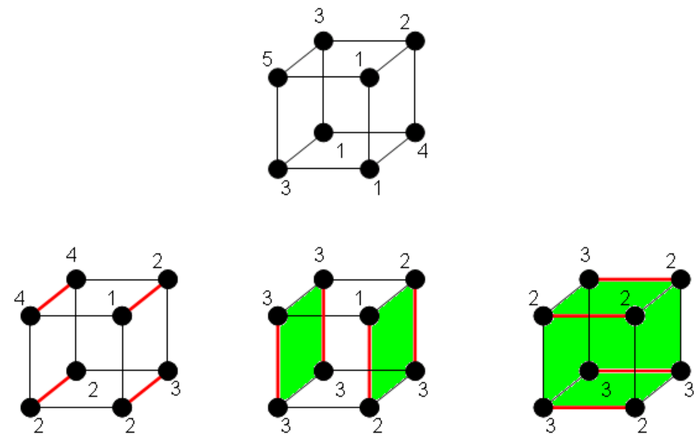


(* algorithm running on node $b_0 \ldots b_{d-1}$ *)
$my\_id := b_0 \ldots b_{d-1}$;
$\mathcal{T}_{my\_id}$:=tokens at this node;
for $i := 0$ to $d-1$ **do**
    $buddy\_id := b_0 \ldots \overline{b_i} \ldots b_{d-1}$;
    SEND $|\mathcal{T}_{my\_id}|/2$ tokens to node $buddy\_id$;
    update $\mathcal{T}_{my\_id}$ accordingly;
    $\mathcal{T}_{buddy\_id}$:=REVC tokens from node $buddy\_id$;
    $\mathcal{T}_{my\_id} := \mathcal{T}_{my\_id} \cup \mathcal{T}_{buddy\_id}$;
end for



Dynamic Token Distribution Algorithm (2)

- Problem 1: Peers are not fractional!



- However, by induction, the integer discrepancy is at most *d* larger than the fractional discrepancy.

$$
\begin{aligned}
|v|_{t+1}^{int} &\leq \left\lceil \frac{|v|_t^{int} + |u|_t^{int}}{2} \right\rceil \leq \left\lceil \frac{\left\lfloor |v|_t^{frac} + \frac{i}{2} \right\rfloor + \left\lfloor |u|_t^{frac} + \frac{i}{2} \right\rfloor}{2} \right\rceil \\
&\leq \frac{\left\lfloor |v|_t^{frac} + \frac{i}{2} \right\rfloor + \left\lfloor |u|_t^{frac} + \frac{i}{2} \right\rfloor}{2} + \frac{1}{2} \\
&\leq \frac{|v|_t^{frac} + |u|_t^{frac} + i + 1}{2} = |v|_{t+1}^{frac} + \frac{i+1}{2}.
\end{aligned}
$$

Dynamic Token Distribution Algorithm (3)

- Problem 2: An adversary inserts at most *J* and removes at most *L* peers *per step*!

- Fortunately, these dynamic changes are balanced quite fast (geometric series).

$$\underbrace{J_t + \frac{J_{t-1}}{2} + \frac{J_{t-2}}{4} + ... + \frac{J_{t-(d-1)}}{2^{d-1}}}_{< 2J} + \underbrace{\frac{J_{t-d}}{2^d} + \frac{J_{t-(d+1)}}{2^d} + \frac{J_{t-(d+2)}}{2^d} + ...}_{\text{shared by all nodes}}$$

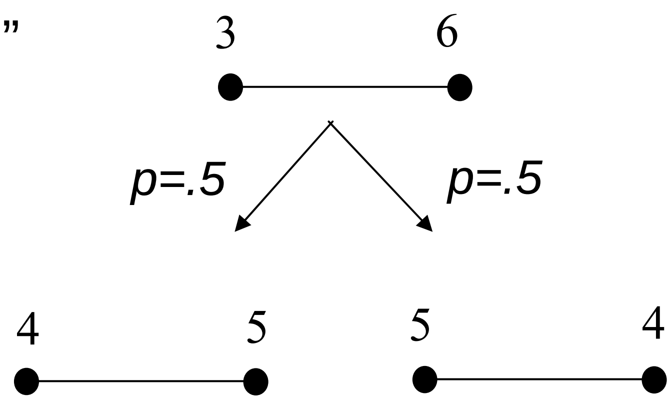
- Thus

Theorem 1: Given adversary *ADV(J,L,1)*, discrepancy never exceeds *2J+2L+d*!

Excursion: Randomized Token Distribution

- Again the static case, but this time assign "dangling" token to one of the edge's vertices *at random*

- "Randomized rounding"



- Dangling tokens are binomially distributed => Chernoff lower tail

Theorem 2: The expected discrepancy is *constant* (~ 3)!

Components: Peer Distribution and Information Aggregation

Peer Distribution

- Goal: Distribute peers evenly among all hypercube nodes in order to balance biased adversarial churn
- Basically a token distribution problem

Counting the total number of [illegible] (information aggregation)

- Goal: Estimate the [illegible]mber of peers in the system and adapt the dimension [illegible]ngly

Tackled next!

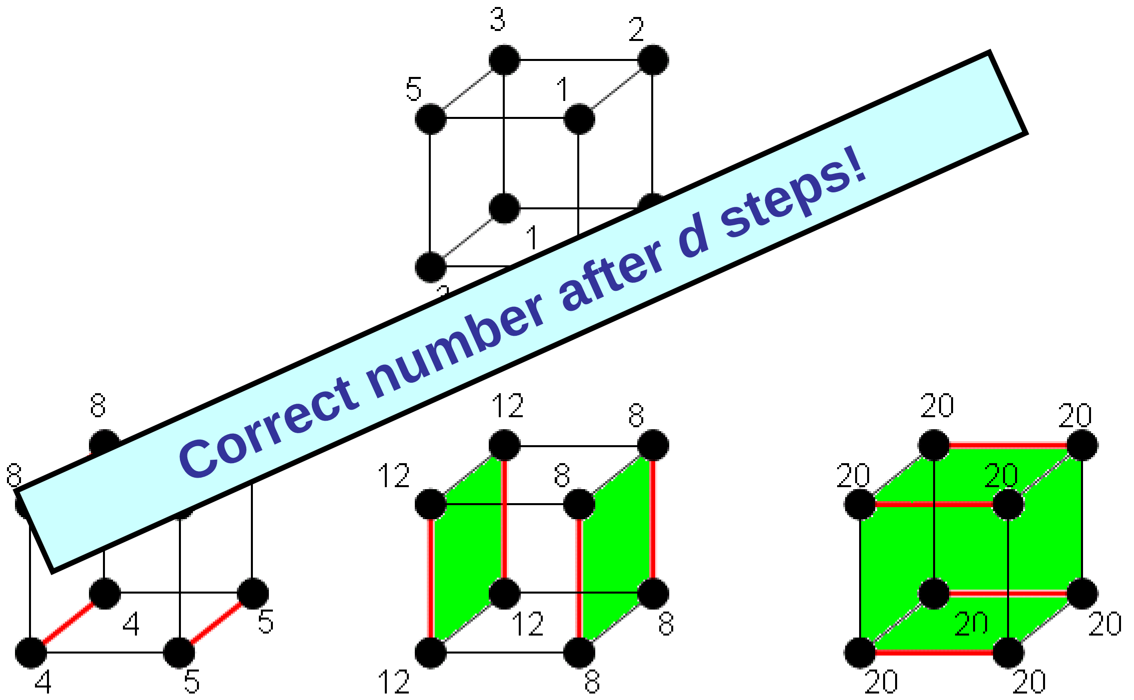
Information Aggregation Algorithm (1)

- Goal: Provide the same (and good!) estimation of the total number of peers presently in the system to all nodes
  - Thresholds for expansion and reduction

- Means: Exploit again the recursive structure of the hypercube!

Information Aggregation Algorithm (2)

Algorithm: Count peers in every sub-cube by exchange with corresponding neighbor!



Correct number after *d* steps!

Information Aggregation Algorithm (3)

- But again, we have a concurrent adversary!
- Solution: Pipelined execution!

Theorem 3: The information aggregation algorithm yields the same estimation to all nodes. Moreover, this number represents the correct state of the system *d* steps ago!

Composing the Components

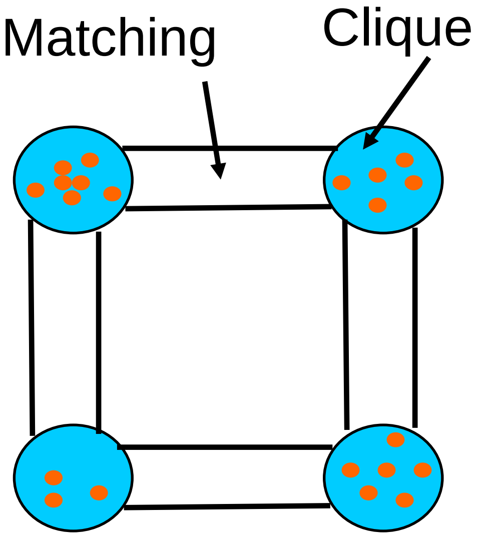
- Our system permanently runs
  - Peer distribution algorithm to balance biased churn
  - Information aggregation algorithm to estimate total number of peers and change dimension accordingly

But: How are peers connected inside a node, and how are the edges of the hypercube represented?

Intra- and Interconnections

- Peers inside the same hypercube vertex are connected *completely* (clique).

- Moreover, there is a matching between the peers of neighboring vertices.



Maintenance Algorithm

- Maintenance algorithm runs in *phases*
  - Phase = 6 rounds

- In phase *i*:
  - Snapshot of the state of the system in round 1
  - One exchange to estimate number of peers in sub-cubes (information aggregation)
  - Balances tokens in dimension *i mod d*
  - Dimension change if necessary

All based on the snapshot made in round 1, ignoring the changes that have happened in-between!

Results for Hypercube Topology

- Given an adversary *ADV(d+1,d+1,6)...*
  => Peer discrepancy at most *5d+4* (Theorem 1)
  => Total number of peers with delay *d* (Theorem 3)

- ... we have, in spite of *ADV(O(log n), O(log n), 1)*:

  – always at least one peer per node,

  – peer degree bounded by *O(log n)* (asymptotically opitmal!),

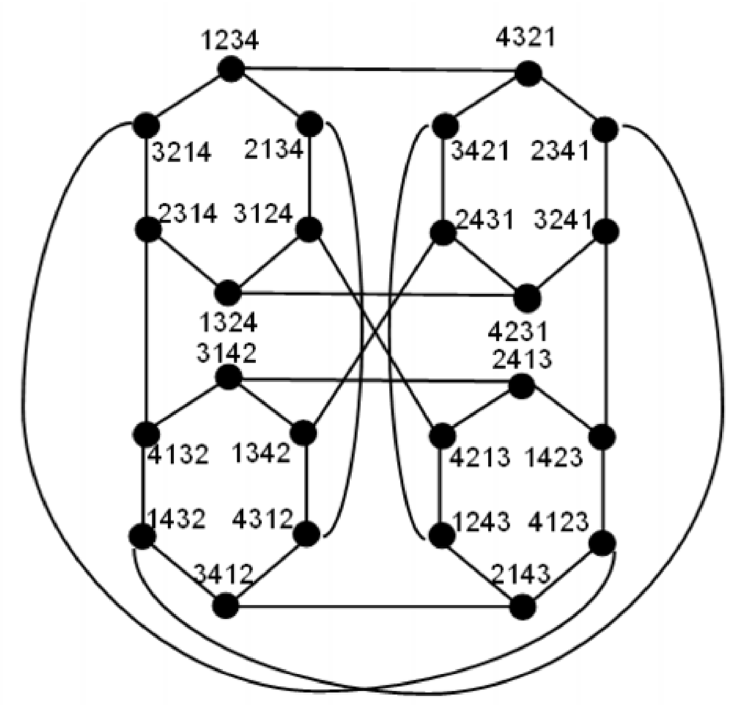
  – network diameter *O(log n).*

A Blueprint for Many Graphs?

- Conclusion: We have achieved an asymptotically optimal fault-tolerance for a *O(log n)* degree and *O(log n)* diameter topology.

- Generalization? We could apply the same tricks for general graphs *G=(V,E)*, given the ingredients (on *G*):
  - –token distribution algorithm
  - –information aggregation algorithm

- For instance: Easy for skip graphs (Δ = *D* = O(*log n*)), possible for pancake graphs (Δ = *D* = *O(log n / loglog n)*).



Open Problems

- Experiences with other graphs?
- Other models for graph dynamics?
- Less messages?

Thank you for your attention!

Discussion

- Questions?
- Inputs?
- Feedback?