Technische Universität München
Fakultät für Informatik
Lehrstuhl für Effiziente Algorithmen
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Assignment 7
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Efficient Algorithms and Datastructures I

Question 1 (10 Points)

The mean M of a set of k integers $\{x_1, x_2, \ldots, x_k\}$ is defined as

$$M = \frac{1}{k}\sum_{i=1}^{k} x_i.$$

We want to maintain a data structure D on a set of integers under the normal INIT, INSERT, DELETE, and FIND operations, as well as a MEAN operation, defined as follows:

1. INIT(D): Create an empty structure D.
2. INSERT(D, x): Insert x in D.
3. DELETE(D, x): Delete x from D.
4. FIND(D, x): Return pointer to x in D.
5. MEAN(D, a, b): Return the mean of the set consisting of elements x in D with $a \leq x \leq b$.

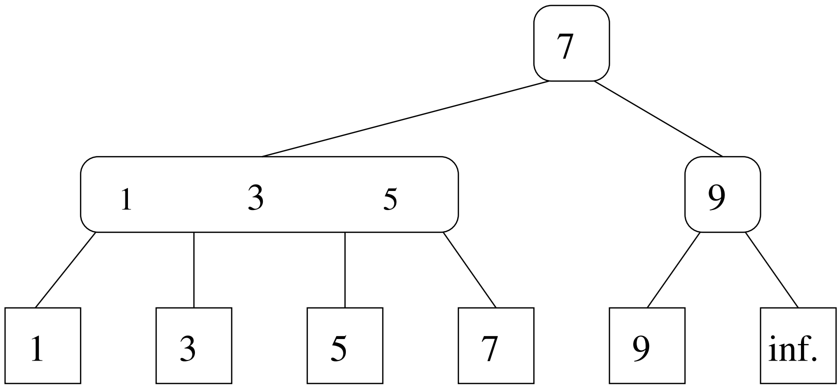
Describe how to modify a standard red-black tree in order to implement D, such that INIT is supported in $O(1)$ time and INSERT, DELETE, FIND, and MEAN are supported in $O(\log n)$ time.

Question 2 (10 Points)

Prove that there exists a sequence of n insert and delete operations on a $(2, 3)$-tree s.t. the total number of split and merge operations performed is $\Omega(n \log n)$.

Question 3 (10 Points)

Carry out the following operations sequentially on the (2,4) tree shown below so that it remains a (2,4) tree and show what the tree looks like after each operation(always carry out each operation on the result of the previous operation):



1. Insert(4)
2. Delete(3)
3. Delete(1)

Question 4 (10 Points)

In double hashing, if we use the hash function $h(k, i) = (h_1(k) + ih_2(k)) \bmod m$, show that when m and $h_2(k)$ have greatest common divisor $d \geq 1$ for some key k, then an unsuccessful search for key k examines $\frac{1}{d}$th of the hash table before returning to slot $h_1(k)$.
(*Note:* When $d = 1$, i.e. when m and $h_2(k)$ are relatively prime, the search may examine the entire hash table.)