Prof. Dr. Susanne Albers
Dr. Dimitrios Letsios
Dario Frascaria
Lehrstuhl für Theoretische Informatik
Fakultät für Informatik
Technische Universität München

Problem set 8
June 7, 2015
Summer Semester 2015

Online and Approximation Algorithms

Due June 15, 2015 before class!

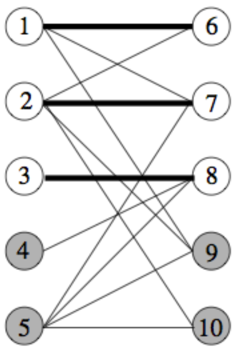
Exercise 1 (Rankings - 10 points)

Consider a bipartite graph $G = (U \cup V, E)$ such that $|U| = |V|$. Moreover, let $\pi(U)$ and $\pi(V)$ be two fixed permutations (rankings) of U and V, respectively. Show that the following methods produce the same matching.

- The vertices in V arrive online according to $\pi(V)$ and vertex $v \in V$ is matched to its unmatched neighbor $u \in U$ with the highest rank with respect to $\pi(U)$.
- The vertices in U arrive online according to $\pi(U)$ and vertex $u \in U$ is matched to its unmatched neighbor $v \in V$ with the highest rank with respect to $\pi(V)$.

Exercise 2 (Augmenting Paths - 10 points)

Consider a bipartite graph $G = (U \cup V, E)$ and a matching M of G. A *simple path* of G is a collection of edges $(v_0, v_1), (v_1, v_2), \ldots, (v_{k-1}, v_k)$ where all v_i's are distinct. Such a path can be also represented as $v_0, v_1, \ldots, v_k$. An *alternating path* of G with respect to M is a simple path which alternates between edges in M and edges in $E - M$. An *augmenting path* of G with respect to M is an alternating path in which the first and last vertices are unmatched (i.e. they are not the endpoint of any edge in M).
For example, in the following graph all paths 4,8,3 and 6,1,7,2 as well as 5,7,2,6,1,9 are alternating with respect to the matching $M = \{(1,6),(2,7),(3,8)\}$. However, only the last one is augmenting.



- Show that a matching is maximum if and only if there are no augmenting paths w.r.t. it.

Exercise 3 (Power-Down Mechanisms, Optimal Offline Cost - 10 points)

Recall the energy efficiency problem presented in class in which there is a system with an active state s_0 and several lower-power sleep states $s_1, s_2, \ldots, s_\ell$. Each state s_i has an individual power-consumption rate r_i and there is a cost $d_{i,j}$ for transitioning from state s_i to state s_j where the transition costs satisfy the triangle inequality. For a maximal idle period of length t, we denote by $OPT(t)$ the optimal offline cost of this period. Show that $OPT(t)$ is a continuous and concave function of t.

Exercise 4 (Lower Envelope Algorithm with Additive Cost - 10 points)

Recall the *Lower Envelope* algorithm presented in class for the online problem of minimizing the energy consumption during idle periods of a computing system equipped with multiple sleep states. This algorithm was shown to be $(3+2\sqrt{2})$-competitive for arbitrary systems. Let $d_{i,j}$ be the cost for transitioning from state s_i to state s_j. For the special case of the problem where the power-down costs are additive, i.e. $d_{i,j} + d_{j,k} = d_{i,k}$ for $i < j < k$, show that the Lower Envelope algorithm is 2-competitive.