

Technique 3: The Primal Dual Method

The previous two rounding algorithms have the disadvantage that it is necessary to solve the LP. The following method also gives an f -approximation without solving the LP.

For estimating the cost of the solution we only required two properties.

1. The solution is dual feasible and, hence,

$$\sum_u y_u \leq \text{cost}(x^*) \leq \text{OPT}$$

where x^* is an optimum solution to the primal LP.

2. The set I contains only sets for which the dual inequality is tight.

Of course, we also need that I is a cover.

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Algorithm 1 PrimalDual

- 1: $y \leftarrow 0$
- 2: $I \leftarrow \emptyset$
- 3: **while** exists $u \notin \bigcup_{i \in I} S_i$ **do**
- 4: increase dual variable y_u until constraint for some
 new set S_ℓ becomes tight
- 5: $I \leftarrow I \cup \{\ell\}$