Technische Universität München
Fakultät für Informatik
Lehrstuhl für Effiziente Algorithmen
Prof. Dr. Harald Räcke
Chris Pinkau

Winter Semester 2014/15
Problem Set 09
December 8th, 2014

Parallel Algorithms

Due date: December 13th, 2014 **before** *class!*

Problem 1 (10 Points)

Given the n-dimensional hypercube, find and prove the following:

(i) the number of vertices,

(ii) the number of edges,

(iii) the diameter,

(iv) the bisection width (the bisection width is the minimal number of edges which have to be cut to create two networks with 2^{n-1} vertices each).

Problem 2 (10 Points)

How many disjoint s-dimensional hypercubes are contained in an r-dimensional cube for $r \geq s$? (For example, a two-dimensional cube contains two one-dimensional cubes.)

Problem 3 (10 Points)

Define a bisection as a cut of a graph, i.e. a subset of nodes or edges, such that the graph is partitioned into two equally sized parts.
Given an d-dimensional hypercube, show that the removal of the nodes with size $\lceil \frac{d}{2} \rceil$ and size $\lfloor \frac{d}{2} \rfloor$ (i.e. nodes with that many 1s) results in a bisection containing $\Theta\left(\frac{2^d}{\sqrt{d}}\right)$ nodes.

Problem 4 (10 Points)

Let u and v be nodes of the d-dimensional hypercube, and let $u_1, u_2, \ldots, u_d$ and $v_1, v_2, \ldots, v_d$ denote their neighbors, respectively. Let π be any permutation on $\{1, 2, \ldots, d\}$. Show that there is an automorphism of the hypercube σ such that $\sigma(u) = v$ and $\sigma(u_i) = v_{\pi(i)}$ for $1 \leq i \leq d$.
Hint: An *automorphism* of a graph is a one-to-one mapping of the nodes to the nodes such that edges are mapped to edges.