Technische Universität München
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Winter Semester 2014/15
Problem Set 07
November 24th, 2014

Parallel Algorithms

Due date: December 1st, 2014 **before** *class!*

Problem 1 (10 Points)

Given a set of positive integers $x_1, \ldots, x_n$ stored in the first n cells of the global memory of an arbitrary CRCW PRAM, the *element distinctness problem* is to determine whether there exist $i \neq j$ such that $x_i = x_j$. Show how to solve this problem in $\mathcal{O}(1)$ time, using n processors.
Hint: Exploit the fact that the x_i's are positive integers.

Problem 2 (10 Points)

Consider the computation of x^n, where x is an input data stored in a location of the global memory.

1. Show that x^n can be computed in one step on the ideal PRAM model.
2. Show that $\Omega(\log n)$ steps are required by the standard arithmetic CREW PRAM. Assume that each step consists of a read instruction, or a write instruction, or an arithmetic operation from $\{+, -, \times, \div\}$.
   Hint: Show that any function computed at the kth step is of degree $\leq 2^k$.

Problem 3 (10 Points)

Recall that there exists a sorting network that sorts n elements with depth $\mathcal{O}(\log n)$ and size $\mathcal{O}(n \log n)$, namely the AKS network.
Using the AKS sorting network, show that sorting n elements can be done in $\mathcal{O}\left(\frac{\log n}{\log(1+\frac{p}{n})}\right)$ parallel steps on the parallel comparison tree model of degree $p \geq n$.
Hint: Shrink each $t = \frac{1}{2} \log\left(1 + \frac{p}{n}\right)$ steps into a single step on the parallel comparison tree model.