Technische Universität München
Fakultät für Informatik
Lehrstuhl für Effiziente Algorithmen
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Winter Semester 2014/15
Problem Set 05
November 10th, 2014

Parallel Algorithms

Due date: November 17th, 2014 **before** *class!*

Problem 1 (10 Points)

1. Using an $\mathcal{O}(\log\log n)$ algorithm to compute the prefix (or suffix) minima of A, design an $\mathcal{O}(\log\log n)$ time algorithm for the range-minima problem using $\mathcal{O}(n\log n)$ operations.
2. Divide the array into subarrays to make this algorithm optimal, i.e. only $\mathcal{O}(n)$ operations must be used.

Problem 2 (20 Points)

Consider the ANSV problem, defined on Problem Set 3.

1. Using a balanced binary tree, develop an $\mathcal{O}\left(\log^2 n\right)$ time algorithm to solve the ANSV problem of an array of length n with a total of $\mathcal{O}(n\log n)$ operations.
   Hint: Use recursion and the developed algorithms for prefix and suffix minima.
2. How can this algorithm run in $\mathcal{O}(\log n)$ time?

Problem 3 (10 Points)

Consider the fast merging algorithm presented in the lecture for merging two sorted sequences of lengths n and m, respectively, with $m \leq n$, in $\mathcal{O}(\log\log m)$ time requiring $\mathcal{O}((n+m)\log\log m)$ work (so not the optimal merging algorithm). Show how to handle the corresponding processor allocation problem to implement this algorithm in a CREW PRAM with $n+m$ processors.
Hint: Store the $j(i)'s$ in a separate array. The pair (B_i, A_i) should be handled by processors $P_{i\sqrt{m}+j(i)+1}, \ldots, P_{(i+1)\sqrt{m}+j(i+1)}$. Decompose the $n+m$ processors into groups, each with $\sqrt{m}$ processors. Let P_{i_ℓ} be the first processor in the ℓ-th group. The k-th processor in group ℓ can determine whether or not P_{i_ℓ} is assigned to the subproblem (B_k, A_k).