

# Project Selection

## Project selection problem:

- ▶ Set  $P$  of possible projects. Project  $v$  has an associated profit  $p_v$  (can be positive or negative).
- ▶ Some projects have requirements (taking course EA2 requires course EA1).
- ▶ Dependencies are modelled in a graph. Edge  $(u, v)$  means “can’t do project  $u$  without also doing project  $v$ .”
- ▶ A subset  $A$  of projects is **feasible** if the prerequisites of every project in  $A$  also belong to  $A$ .

Goal: Find a feasible set of projects that maximizes the profit.

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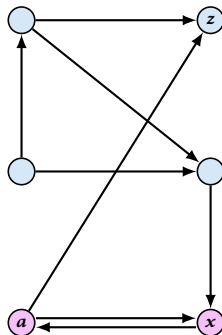
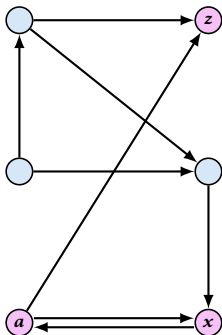
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## The prerequisite graph:

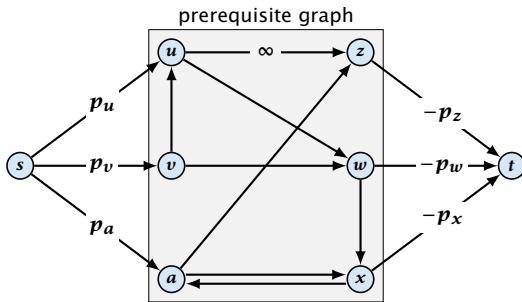
- ▶  $\{x, a, z\}$  is a feasible subset.
- ▶  $\{x, a\}$  is infeasible.



# Project Selection

## Mincut formulation:

- ▶ Edges in the prerequisite graph get infinite capacity.
- ▶ Add edge  $(s, v)$  with capacity  $p_v$  for nodes  $v$  with positive profit.
- ▶ Create edge  $(v, t)$  with capacity  $-p_v$  for nodes  $v$  with negative profit.



## Theorem 2

$A$  is a mincut if  $A \setminus \{s\}$  is the optimal set of projects.

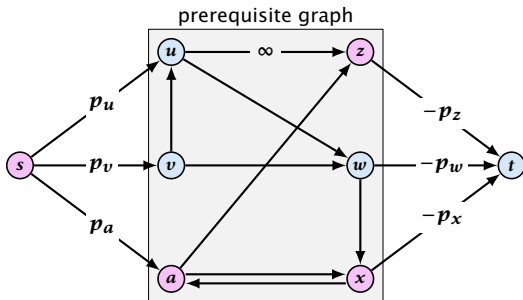


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**Proof.**

- ▶  $A$  is feasible because of capacity infinity edges.

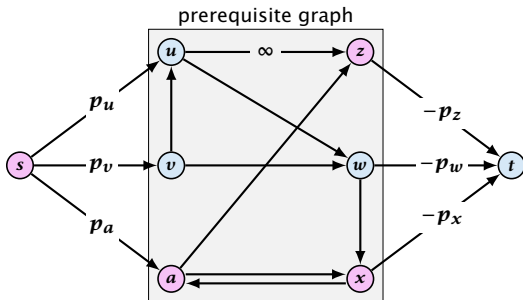


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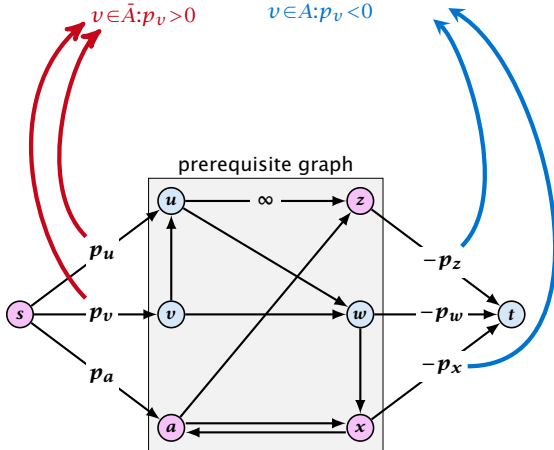
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$$= \sum_{v: p_v > 0} p_v - \sum_{v \in A} p_v$$

