Lemma 10.3 *Let* $\varphi = a \cdot x \leq b$ *and* $s = \sum_{i=1}^{k} |a_i|$. *All states* s_j *added to the worklist during the execution of PAtoDFA*(φ) *satisfy*

$$-|b| - s \leq j \leq |b| + s.$$

Proof: The property holds for s_b, the first state added to the worklist. We show that if all the states added to the worklist so far satisfy the property, then so does the next one.

Let s_j be this next state. Then there exists a state s_k in the worklist and $\zeta \in \{0,1\}^n$ such that $j = \lfloor \frac{1}{2}(k - a \cdot \zeta) \rfloor$. Since by assumption s_k satisfies the property we have

$$-|b| - s \leq k \leq |b| + s$$

and so

$$\left\lfloor \frac{-|b| - s - a \cdot \zeta}{2} \right\rfloor \leq j \leq \left\lfloor \frac{|b| + s - a \cdot \zeta}{2} \right\rfloor \qquad (10.2)$$

Now we observe

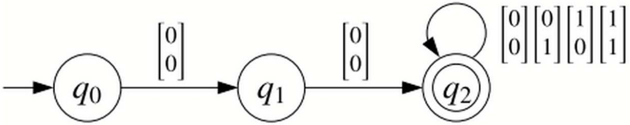
$$
\begin{array}{rcccl}
-|b| - s & \leq & \dfrac{-|b| - 2s}{2} & \leq & \left\lfloor \dfrac{-|b| - s - a \cdot \zeta}{2} \right\rfloor \\
\left\lfloor \dfrac{|b| + s - a \cdot \zeta}{2} \right\rfloor & \leq & \dfrac{|b| + 2s}{2} & \leq & |b| + s
\end{array}
$$

which together with 10.2 yields

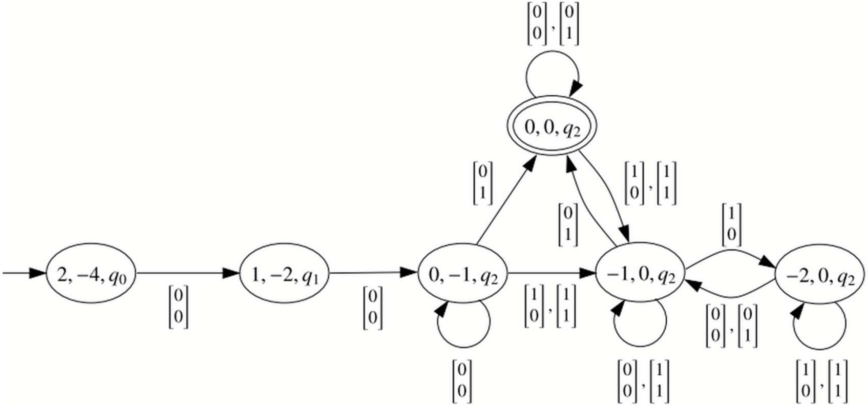
$$-|b| - s \leq j \leq |b| + s$$

and we are done. $\square$

$$\exists z\, x = 4z \;\wedge\; \exists w\, y = 4w \;\wedge\; 2x - y \leq 2 \;\wedge\; x + y \geq 4$$



DFA for the formula $\exists z x = 4z \;\wedge\; \exists w\, y = 4w$.


$0, 0, q_2$
$2, -4, q_0$
$1, -2, q_1$
$0, -1, q_2$
$-1, 0, q_2$
$-2, 0, q_2$