Technische Universität München
Fakultät für Informatik
Lehrstuhl für Effiziente Algorithmen (LEA)
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Spring Semester 2014
Problem set 12
July 02, 2014

Online and approximation algorithms

Due July 09, 2014 before class!

Exercise 1 (MAX-NAE3SAT - 10 points)

We consider the *MAX-NAE3SAT* (Max-not-all-equal-3SAT) problem. In this problem we are given a *3CNF* formula and our goal is to find a variable assignment, such that the number of clauses containing at least one true **and** one false literal is maximized.
Develop an approximation algorithm for *MAX-NAE3SAT* with an expected approximation factor of $\frac{3}{4}$.

Exercise 2 (Fractional Knapsack - 10 points)

Consider the fractional Knapsack problem: Instead of only being able to pack an item completely or not at all we allow fractions of items to be packed.
Develop an exact algorithm for the fractional Knapsack problem and show that it runs in $\mathcal{O}(n)$ expected time.

Exercise 3 (2SAT - 10 points)

Consider the *2SAT* problem. In this problem we are given a boolean formula in *2CNF* and have to decide whether the formula is satisfiable or not.

(a) Describe a method of encoding the given *2CNF*-formula as a directed graph.

(b) Develop an algorithm that decides in polynomial time whether the given formula can be satisfied or not.

(c) Develop an algorithm that returns a satisfying assignment if it exists and show that it runs in polynomial time as well.

Exercise 4 (Quiztime - 10 points)

Decide for each statement whether it is true or false and give a brief explanation (1-2 sentences).

1. FIFO is $(\log k)$-competitive.
2. Given an online problem, the competitive ratio of the best randomized online algorithm against any oblivious adversary is equal to the competitive ratio of the best deterministic online algorithm under a worst-case input distribution.
3. There is a deterministic $\sqrt{2}$-competitive algorithm for the list update problem.
4. Given an online problem, if A is 4-competitive against adaptive online adversaries and there exists a 2-competitive algorithm against oblivious adversaries, then there exists a 6-competitive algorithm against adaptive offline adversaries.
5. Every deterministic algorithm for the wall problem is $\Omega(\sqrt{n})$-competitive.
6. The competitive ratio of the greedy bipartite matching algorithm is at least $\frac{3}{4}$.
7. The Lower Envelope Algorithm (LEA) achieves a competitive ratio of 7 for general state systems.
8. Metrical task systems are a generalization of the paging problem.
9. There is a 1.5-approximation for the Traveling Salesman problem (TSP).
10. The Knapsack problem is $\mathcal{NP}$-complete.