Technische Universität München
Fakultät für Informatik
Lehrstuhl für Effiziente Algorithmen (LEA)
Prof. Dr. Susanne Albers
Moritz Fuchs

Spring Semester 2014
Problem set 6
May 14, 2014

Online and approximation algorithms

Due May 21, 2014 before class!

Exercise 1 (Burrows-Wheeler-Transformation - 10 points)

(a) Encode the string 'ananas' using the Burrows-Wheeler-transformation.

(b) Decode (mrbnnaaaaa , 5) using the linear time method discussed in the lecture. Include intermediate steps!

Exercise 2 (Huffman-code - 10 points)

(a) Encode the string 'assassin' using the Huffman-code. Include the resulting tree as well as the encoding table.

(b) Decode the encoded string from (a). Include intermediate steps.

Exercise 3 (Huffman-Code II - 10 points)

Let Q be a memoryless source over the alphabet $\Sigma = \{x_1, ..., x_n\}$ using the probability vector $P = (p_1, ..., p_n)$ with $p_1 \leq ... \leq p_n$. For a given code K the expected number of bits for encoding a symbol is $B(K) = \sum_{i=1}^{n} p_i l(x_i)$ where $l(x_i)$ is the number of bits needed to encode x_i.

Show that there exists an optimal binary prefix code in which the encoded symbols x_1 and x_2 have the same length and differ only in the last bit.

Exercise 4 (Game tree evaluation - 10 points)

A binary game tree is a complete binary tree of height $2k$. Each internal node at even distance from the root is labeled with AND, each internal node at odd distance from the root is labeled with OR. Each leaf has value 0 or 1. When evaluating the tree, every AND-node returns the smallest value of its children, every OR-node returns the largest value of its children.

The game tree evaluation problem is the problem of computing the value of the root node. The input we are given is the value of the 4^k leaf-nodes. The cost of an algorithm for this problem is defined as the number of accesses to leaf-nodes.

(a) Show that each AND/OR can be replaced by NOR without altering the value of the root node.

(b) Use Yao's principle to show a lower bound on the cost of randomized (Las Vegas) algorithms for the game tree evaluation problem.

Hint 1: Tarsi [1] showed in 1983 that for a subtree with root v and height h, the expected cost for evaluating v is minimal if the value of one of the children of v is evaluated before one inspects any leaf in the subtree rooted at the other child. Hence if the first child evaluates to 1, an optimal algorithm skips the second child.

Hint 2: Choose a distribution over the inputs s.t. each internal node has the same distribution as the input values.

Literatur

[1] Michael Tarsi. Optimal search on some game trees. *J. ACM*, 30(3):389–396, July 1983.