Technische Universität München
Fakultät für Informatik
Lehrstuhl für Effiziente Algorithmen
Prof. Dr. Ernst W. Mayr
Chris Pinkau

Spring Semester 2014
Problem Set 7
June 2, 2014

Complexity Theory

Due date: June 12, 2014 **before** *class!*

Problem 1 (10 Points)

Prove that the following language is **PSPACE**-complete:
In-Place Acceptance: Given a Turing machine M and an input x, does M accept x without ever leaving the first $|x| + 1$ symbols on its strings?

Problem 2 (10 Points)

Recall the definition of alternating Turing machines (ATM) with control states partitioned into sets $Q_\forall$ and $Q_\exists$, and the corresponding class **AP**.

1. Show that a language $L \in \mathbf{AP}$ decided by an *existential* ATM (i.e. $Q_\forall = \emptyset$) is in $\mathcal{NP}$.
2. Show that a language $L \in \mathbf{AP}$ decided by an *universal* ATM (i.e. $Q_\exists = \emptyset$) is in co$\mathcal{NP}$.
3. Show that $\mathbf{AP} = \text{co}\mathbf{AP}$.
4. Show that **PSPACE** is contained in **AP** by showing that tQbf $\in \mathbf{AP}$.

Problem 3 (10 Points)

Prove $\mathbf{AL} = \mathcal{P}$.

Problem 4 (10 Points)

A language L is called *downward self-reducible* if it can be solved by a polynomial-time oracle Turing machine with an oracle for L, however in such a way that on input $x \in \{0, 1\}^n$ the machine only asks queries of length *strictly less* than n.
Prove that if L is downward self-reducible, then $L \in \mathbf{PSPACE}$.