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Problem Set 4
May 12, 2014

Complexity Theory

Due date: May 19, 2014 **before** *class!*

Problem 1 (10 Points)

1. Assume $A \preceq_m^p B$. Show that then also $\overline{A} \preceq_m^p \overline{B}$.
2. Show that if a complexity class $\mathcal{C}$ is closed under $\preceq_m^p$, then so is co$\mathcal{C}$.
3. Show that co$\mathcal{NP}$ is closed under union and intersection.

Problem 2 (10 Points)

Define the following two covering problems:

- A *vertex cover* of a graph $G = (V, E)$ is a set of vertices $V' \subseteq V$, where every edge in E is incident to at least one vertex in V'.
  Let Vertex Cover $= \{(G, k) : G$ has a vertex cover of size at most $k\}$.
- Given a set U, and a family S of subsets of U, a *set cover* of U is a subfamily of sets $C \subseteq S$ whose union is U.
  Let Set Cover $= \{(U, S, k) : U$ has a set cover of size at most $k\}$.

Show the following two claims.

1. Vertex Cover is $\mathcal{NP}$-complete.
2. Set Cover is $\mathcal{NP}$-complete.

Problem 3 (10 Points)

Define a *regular expression* r over $\{0, 1\}$ as

$$r ::= 0 \mid 1 \mid rr \mid (r|r),$$

or, equivalently,

$$\begin{aligned} r &\to 0 \\ r &\to 1 \\ r &\to rr \\ r &\to (r|r). \end{aligned}$$

The problem RegExpEq is about the question whether two languages defined by two different regular expressions are identical. A special case of this is the language RegExpEq_*, which is defined as

$$\text{RegExpEq}_* = \{r : \text{there exists an } n \in \mathbb{N} \text{ s.t. } L(r) = \Sigma^n\},$$

where $L(r)$ denotes the language generated by r, i.e., the set of all words that can be generated by using the rules of r.
Given $\Sigma = \{0, 1\}$, show that RegExpEq_* is co$\mathcal{NP}$-complete.

Problem 4 (10 Points)

Define the class $\mathbf{DP} = \{L = L_1 \cap L_2 : L_1 \in \mathcal{NP}, L_2 \in \text{co}\mathcal{NP}\}$. (Note that we do not know if $\mathbf{DP} = \mathcal{NP} \cap \text{co}\mathcal{NP}$.) Consider the following languages:

$\text{ExactIndset} = \{(G, k) : \text{ the largest independent set of } G \text{ has size exactly } k\}$,
$\text{Critical Sat} = \{\varphi : \varphi \text{ in 3CNF is unsatisfiable, but deleting any clause makes it satisfiable}\}$.

Show the following:

1. ExactIndset $\in$ **DP**.
2. Critical Sat is **DP**-complete.
   Hint: Use a **DP**-complete problem and reduce it to Critical Sat. What would be the obvious choice for a **DP**-complete problem?