Technische Universität München
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Problem Set 2
April 28, 2014

Complexity Theory

Due date: May 5, 2014 **before** *class!*

Problem 1 (10 Points)

(i) One can easily show that the polynomial-time many-to-one reduction $\preceq_m^p$ is reflexive (i.e. $A \preceq_m^p A$ for all languages A) and transitive (i.e., if $A \preceq_m^p B$ and $B \preceq_m^p C$, then $A \preceq_m^p C$). But is it also commutative (i.e., if $A \preceq_m^p B$, then $B \preceq_m^p A$)?

(ii) Show or disprove: $\mathcal{NP}$ is closed under union or intersection, respectively. (Meaning that if $L_1, L_2 \in \mathcal{NP}$, then $L_1 \cup L_2 \in \mathcal{NP}$ or $L_1 \cap L_2 \in \mathcal{NP}$, respectively.)

Problem 2 (10 Points)

Define the following problems:

- DNF-Sat is the set of all satisfiable boolean formulae in disjunctive normal form.
- 2Sat is the set of all satisfiable boolean formulae in conjunctive normal form where every clause consists of at most two literals.

(i) Prove that DNF-Sat is in $\mathcal{P}$.

(ii) Prove that 2Sat is in $\mathcal{P}$.

Problem 3 (10 Points)

Let Binary LP be the set of satisfiable integer linear programs with solutions in $\{0, 1\}$. Prove that 3Sat $\preceq_m^p$ Binary LP. Show how your reduction works on the formula

$$(x \vee \overline{y} \vee \overline{z}) \wedge (x \vee y \vee \overline{w}) \wedge (\overline{x} \vee y \vee \overline{w}) \wedge (\overline{y} \vee z \vee w).$$

Problem 4 (10 Points)

(Berman 1978) A *unary* language contains strings of the form 1^m, i.e. strings of m ones for some $m > 0$. Show that if a $\mathcal{NP}$-complete unary language exists, then $\mathcal{P} = \mathcal{NP}$.