Technische Universität München
Fakultät für Informatik
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Winter Semester 2013/14
Problem Set 5
November 19th, 2013

Parallel Algorithms

Due date: November 27th, 2013 **before** *class!*

Problem 1 (10 Points)

1. Show that the odd-even merge algorithm works correctly.
2. Formally derive the odd-even merge sorting network and its depth and size.

Problem 2 (10 Points)

Recall: A binary sequence is *bitonic* if it is a concatenation of two subsequences such that one is monotonically increasing and the other is monotonically decreasing, or vice versa. A sequence $X = (x_0, \dots, x_{n-1})$ is *bitonic* if, for some $j < n$, we have

$$x_{j \bmod n} \leq x_{(j+1) \bmod n} \leq \cdots \leq x_{\ell \bmod n} \quad \text{and}$$
$$x_{(\ell+1) \bmod n} \geq x_{(\ell+2) \bmod n} \geq \cdots \geq x_{(j+n-1) \bmod n}$$

for some ℓ. That is, the circle $x_0 \to x_1 \to \cdots \to x_{n-1} \to x_0$ can be partitioned into two monotonic parts.

Show the zero-one principle for bitonic sequences: An n-input comparator network is a bitonic merging network if and only if it merges correctly all binary bitonic sequences of length n.

Problem 3 (10 Points)

Show that a bitonic merging network can be constructed as follows:

- Given a bitonic sequence, merge $(x_1, x_3, x_5, \dots)$ and $(x_2, x_4, x_6, \dots)$ in bitonic mergers whose lines are interleaved,
- compare and interchange the outputs in pairs, beginning with the least significant pairs.

Problem 4 (10 Points)

Recall that there exists a sorting network that sorts n elements with depth $\mathcal{O}(\log n)$ and size $\mathcal{O}(n \log n)$, namely the AKS network.

Using the AKS sorting network, show that sorting n elements can be done in $\mathcal{O}\left(\frac{\log n}{\log(1+\frac{p}{n})}\right)$ parallel steps on the parallel comparison tree model of degree $p \geq n$.

Hint: Shrink each $t = \frac{1}{2} \log\left(1 + \frac{p}{n}\right)$ steps into a single step on the parallel comparison tree model.