Efficient Algorithms and Datastructures I

Question 1 (10 Points)

Solve the following recurrence using a generating function:

$$a_n = a_{n-1} + a_{n-2} \text{ for } n \geq 2 \text{ with } a_0 = 0 \text{ and } a_1 = 1.$$

Question 2 (10 Points)

Solve the following recurrence using a generating function:

$$a_n = 5a_{n-1} - 8a_{n-2} + 4a_{n-3} \text{ for } n \geq 3 \text{ with } a_0 = 1, a_1 = 3 \text{ and } a_2 = 11.$$
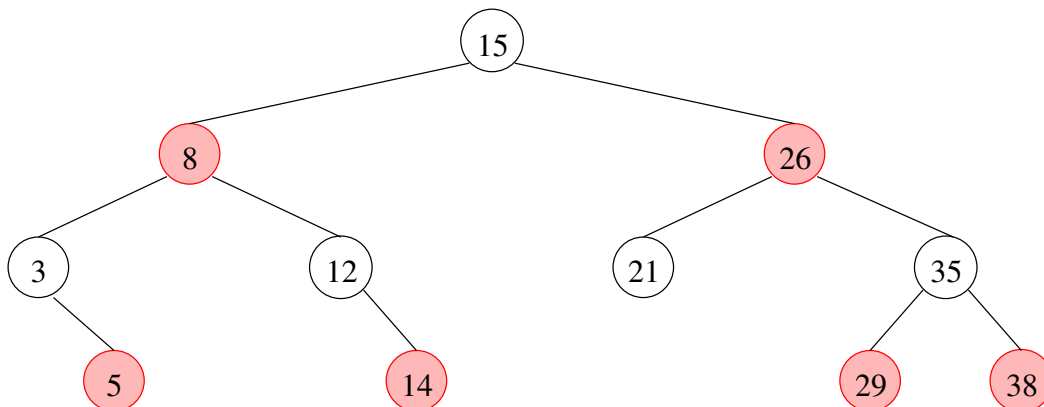
Question 3 (10 Points)

Give tight asymptotic bounds for $T(n)$:

$$T(n) = 2T\left(\frac{n}{2}\right) + \frac{n}{\log n}$$

Question 4 (10 Points)

Carry out the following operations sequentially on the red-black tree shown below so that it remains a red-black tree and show what the tree looks like after each operation(always carry out each operation on the result of the previous operation):

1. Insert 10

2. Delete 29

3. Delete 21

4. Delete 3