Complexity Theory

Due date: April 30, 2013 **before** class!

Problem 1 (10 Points)

(i) One can easily show that the polynomial-time many-to-one reduction $\preceq_m^p$ is reflexive (i.e. $A \preceq_m^p A$ for all languages A) and transitive (i.e., if $A \preceq_m^p B$ and $B \preceq_m^p C$, then $A \preceq_m^p C$). But is it also commutative (i.e., if $A \preceq_m^p B$, then $B \preceq_m^p A$)?

(ii) Show or disprove: $\mathcal{NP}$ is closed under union or intersection, respectively. (Meaning that if $L_1, L_2 \in \mathcal{NP}$, then $L_1 \cup L_2 \in \mathcal{NP}$ or $L_1 \cap L_2 \in \mathcal{NP}$, respectively.)

Problem 2 (10 Points)

Define the following problems:

- DNF-Sat is the set of all satisfiable boolean formulae in disjunctive normal form.

- 2Sat is the set of all satisfiable boolean formulae in conjunctive normal form where every clause consists of at most two literals.

(i) Prove that DNF-Sat is in $\mathcal{P}$.

(ii) Prove that 2Sat is in $\mathcal{P}$.

Problem 3 (10 Points)

Let Tautology $= \{\varphi : \varphi$ is a boolean formula that is satisfied by every assignment$\}$.
Show that $\mathcal{NP} = $ co-$\mathcal{NP}$ if and only if 3Sat and Tautology are polynomial-time reducible to one another.

Problem 4 (10 Points)

(Berman 1978) A *unary* language contains strings of the form 1^m, i.e. strings of m ones for some $m > 0$. Show that if a $\mathcal{NP}$-complete unary language exists, then $\mathcal{P} = \mathcal{NP}$.