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Efficient Algorithms and Datastructures II

Aufgabe 1 (10 Punkte)

(a) Convince yourself that the following LP solves the max-flow problem:

$$\begin{array}{lrcll} \text{maximize} & \sum_{p \in P} x_p & & & \\ \text{subject to} & \sum_{p \ni e} x_p & \leq & c_e & \forall e \in E \\ & x_p & \geq & 0 & \end{array}$$

where c_e denotes the capacity (upper bound) of edge e and P is the set of all paths from the source to the target.

(b) Write the dual of the above linear program.

Aufgabe 2 (10 Punkte)

(a) Observe that the dual (and hence the primal) linear program above can be solved efficiently, given a polynomial time seperation oracle for the dual.

(b) Construct a polynomial time seperation oracle for the dual program. (*Hint*: Use an algorithm for solving the shortest path problem as the separation oracle.)

Aufgabe 3 (10 Punkte)

Consider the following linear program:

$$\begin{array}{lrcll} \text{minimize} & \sum_{e \in E} c_e x_e & & & \\ \text{subject to} & x(\delta(v)) & = & 2 & \forall v \in V \\ & x(\delta(U)) & \geq & 2 & \forall U \subset V, 2 \leq |U| \leq |V| - 2 \\ & x_e & \leq & 1 & \forall e \in E \\ & x_e & \geq & 0 & \forall e \in E \end{array}$$

where c_i is the cost of the i-th edge, $\delta(U)$ denotes the set of edges with exactly one endpoint in U and $x(F) = \sum_{f \in F} x_f, \forall F \subseteq E$. Show how to solve this LP by the ellipsoid method. (*Hint*: The min-cut problem can be solved in polynomial time.)