Technische Universität München
Fakultät für Informatik
Lehrstuhl für Effiziente Algorithmen
Prof. Dr. Ernst W. Mayr
Chris Pinkau

Winter Semester 2012/13
Problem Set 3
November 6, 2012

Parallel Algorithms

Due Date: November 13, 2012 **before** *class!*

Problem 1 (10 Points)

The *transitive closure* of a directed graph $G = (V, E)$ is the graph $G^* = (V, E^*)$, where E^* consists of all pairs (i, j) such that either $i = j$ or there exists a directed path from i to j.
The input graph G is given by its incidence matrix A, and the task is to compute the incidence matrix A^* of its transitive closure. Describe a boolean circuit to compute A^*. Assume that A is an $n \times n$ matrix and that $n = 2^p$.

Problem 2 (10 Points)

Given $n = 2^k$ and two n-bit numbers, the task is to add these numbers. Suppose every processor adds only bit-wise.

(i) Describe an approach on how to compute the behavior of the ith carry bit in relation to the $(i-1)$st carry bit.

(ii) Describe how to compute this for all n carry bits in only $O(\log n)$ bit steps.

Problem 3 (10 Points)

Using Problem 2, describe a parallel algorithm for adding two n-bit numbers in $O(\log n)$ steps.

Problem 4 (10 Points)

Derive an algorithm for adding k n-bit numbers using $O(\log k + \log n)$ steps. You may use $k \cdot n$ processors, since the problem has that many inputs.
Hint: First show that the addition of three n-bit numbers can be reduced to the addition of two $(n + 1)$-bit numbers in one step.