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Effiziente Algorithmen und Datenstrukturen I

Aufgabe 1 (10 Punkte)

If we modify the chaining scheme so that each list is kept in sorted order, how does it affect the running time for successful searches, unsuccessful searches, insertions, and deletions?

Aufgabe 2 (10 Punkte)

In double hashing, if we use the hash function $h(k, i) = (h_1(k) + ih_2(k)) \bmod m$, show that when m and $h_2(k)$ have greatest common divisor $d \geq 1$ for some key k, then an unsuccessful search for key k examines $\frac{1}{d}$th of the hash table before returning to slot $h_1(k)$.
(*Note:* When $d = 1$, i.e. when m and $h_2(k)$ are relatively prime, the search may examine the entire hash table.)

Aufgabe 3 (10 Punkte)

1. Consider a bipartite graph with partitions A, B where $|A| = |B| = n$. Let there be $m = \Theta(n)$ edges in this graph, chosen uniformly at random. In this graph, find the expected number of

   (a) 2-cycles.

   (b) 3-cycles.

   (c) 4-cycles.

   (*Note:* If we let n be the size of one hash table and m be the number of keys, then the above question asks for the number of 2-cycles, 3-cycles and 4-cycles in cuckoo hashing where each edge in the graph denotes the 2 hash values of a function)

2. A pseudoforest is an undirected graph in which every connected component has at most one cycle. If a graph G has the property that, for every subset S of its vertices, the number of edges in the induced subgraph of S is at most the number of vertices in S, then G is a pseudoforest. Again, let G be a random bipartite graph with $m = \Theta(n)$ as above. What is the probability that such a bipartite graph has a pseudoforest of size k where k is a constant?