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Effiziente Algorithmen und Datenstrukturen I

Aufgabe 1 (10 Punkte)

Prove the following statements:

1. $\forall c \in \mathbb{R}^+$, $c \cdot f(n) \in \Theta(f(n))$
2. $f(n) + g(n) \in \Omega(f(n))$
3. $f(n) \in O(g(n)) \Rightarrow f(n) + g(n) \in O(g(n))$
4. $f(n) \in o(g(n))$ and $g(n) \in O(h(n)) \Rightarrow h(n) \in \omega(f(n))$
5. $f(n) \in O(g(n))$ and $g(n) \in O(f(n)) \Leftrightarrow f(n) \in \Theta(g(n))$

Aufgabe 2 (10 Punkte)

For constants $c, \epsilon > 0$ and $k > 1$, arrange the following functions of n in non-decreasing asymptotic order so that $f_i(n) = O(f_{i+1}(n))$ for two consecutive functions in your sequence. Also indicate whether $f_i(n) = \Theta(f_{i+1}(n))$ holds or not.

$$n^k, \sqrt{n}, 2^n, n^{1+sin(n)}, \log(n!), n^{k+\epsilon}, n^n, n, n^k(\log n)^c, n!, n\log n, 3^n, n\log\log n, n\log(n^2)$$

Aufgabe 3 (10 Punkte)

Solve the following recurrence relations:

1. $a_n = a_{n-1} + 2^{n-1}$ with $a_0 = 2$.
2. $a_n = a_{n-1} + 8a_{n-2} - 12a_{n-3}$ with $a_0 = -1$, $a_1 = 11$ and $a_2 = -27$.

Aufgabe 4 (10 Punkte)

Given two $n \times n$ matrices A and B where n is a power of 2, we know how to find $C = A \cdot B$ by performing n^3 multiplications. Now let us consider the following approach. We partition A, B and C into equally sized block matrices as follows:

$$A = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} B = \begin{bmatrix} B_{11} & B_{12} \\ B_{21} & B_{22} \end{bmatrix} C = \begin{bmatrix} C_{11} & C_{12} \\ C_{21} & C_{22} \end{bmatrix}$$

Consider the following matrices:

$$
\begin{aligned}
M_1 &= (A_{11} + A_{22})(B_{11} + B_{22}) \\
M_2 &= (A_{21} + A_{22})B_{11} \\
M_3 &= A_{11}(B_{12} - B_{22}) \\
M_4 &= A_{22}(B_{21} - B_{11}) \\
M_5 &= (A_{11} + A_{12})B_{22} \\
M_6 &= (A_{21} - A_{11})(B_{11} + B_{12}) \\
M_7 &= (A_{12} - A_{22})(B_{21} + B_{22})
\end{aligned}
$$

Then,

$$
\begin{aligned}
C_{11} &= M_1 + M_4 - M_5 + M_7 \\
C_{12} &= M_3 + M_5 \\
C_{21} &= M_2 + M_4 \\
C_{22} &= M_1 - M_2 + M_3 + M_6
\end{aligned}
$$

1. Convince yourself that the matrices C_{ij} evaluated as above are indeed correct. Don't write anything to prove this.

2. Design an efficient algorithm for multiplying two $n \times n$ matrices based on these facts. Analyze its running time.