Grundlagen: Algorithmen und Datenstrukturen

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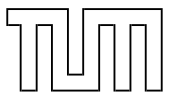
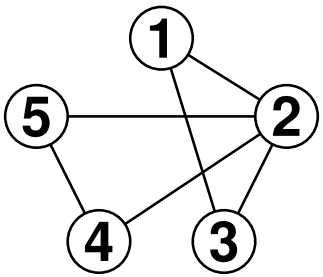
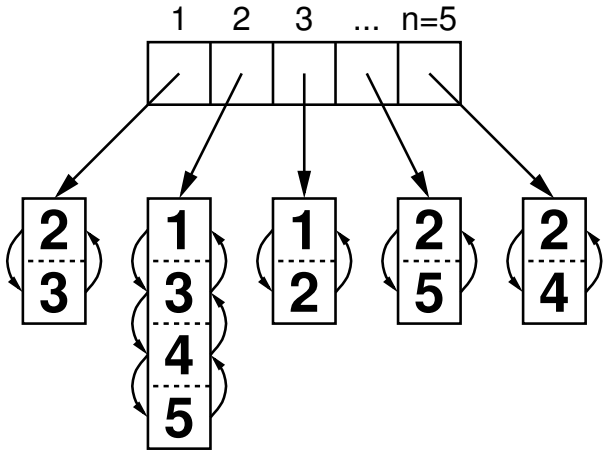


Übersicht

Adjazenzliste





Unterschiedliche Varianten:
einfach/doppelt verkettet, linear/zirkulär

Vorteil:

- Einfügen von Kanten in $\mathcal{O}(d)$ oder $\mathcal{O}(1)$
- Löschen von Kanten in $\mathcal{O}(d)$ (per Handle in $\mathcal{O}(1)$)
- mit unbounded arrays etwas cache-effizienter

Nachteil:

- Zeigerstrukturen verbrauchen relativ viel Platz und Zugriffszeit

Adjazenzliste + Hashtabelle

- speichere Adjazenzliste (Liste von adjazenten Nachbarn bzw. inzidenten Kanten zu jedem Knoten)
- speichere Hashtabelle, die zwei Knoten auf einen Zeiger abbildet, der dann auf die ggf. vorhandene Kante verweist

Zeitaufwand:

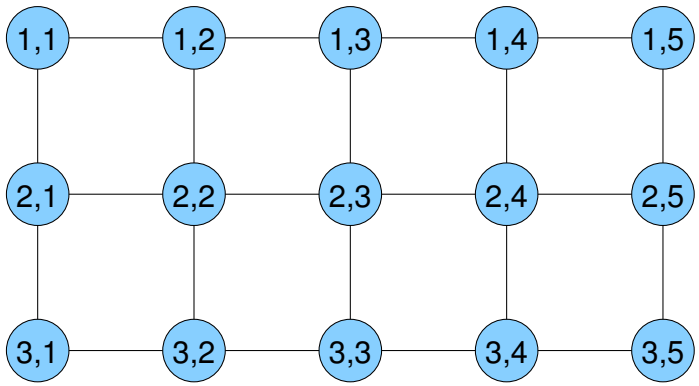
- *G*.find(Key *i*, Key *j*): $\mathcal{O}(1)$ (worst case)
- *G*.insert(Edge *e*): $\mathcal{O}(1)$ (im Mittel)
- *G*.remove(Key *i*, Key *j*): $\mathcal{O}(1)$ (im Mittel)
- Speicheraufwand: $\mathcal{O}(n+m)$

Implizite Repräsentation

Beispiel: Gitter-Graph (grid graph)

- definiert durch zwei Parameter k und ℓ

$$
\begin{aligned}
V &= [1,\ldots,k] \times [1,\ldots,\ell] \\
E &= \Big\{((i,j),(i,j')) \in V^2 : |j-j'| = 1\Big\} \cup \\
&\quad \Big\{((i,j),(i',j)) \in V^2 : |i-i'| = 1\Big\}
\end{aligned}
$$



- Kantengewichte könnten in 2 zweidimensionalen Arrays gespeichert werden:
  eins für waagerechte und eins für senkrechte Kanten

Graphtraversierung

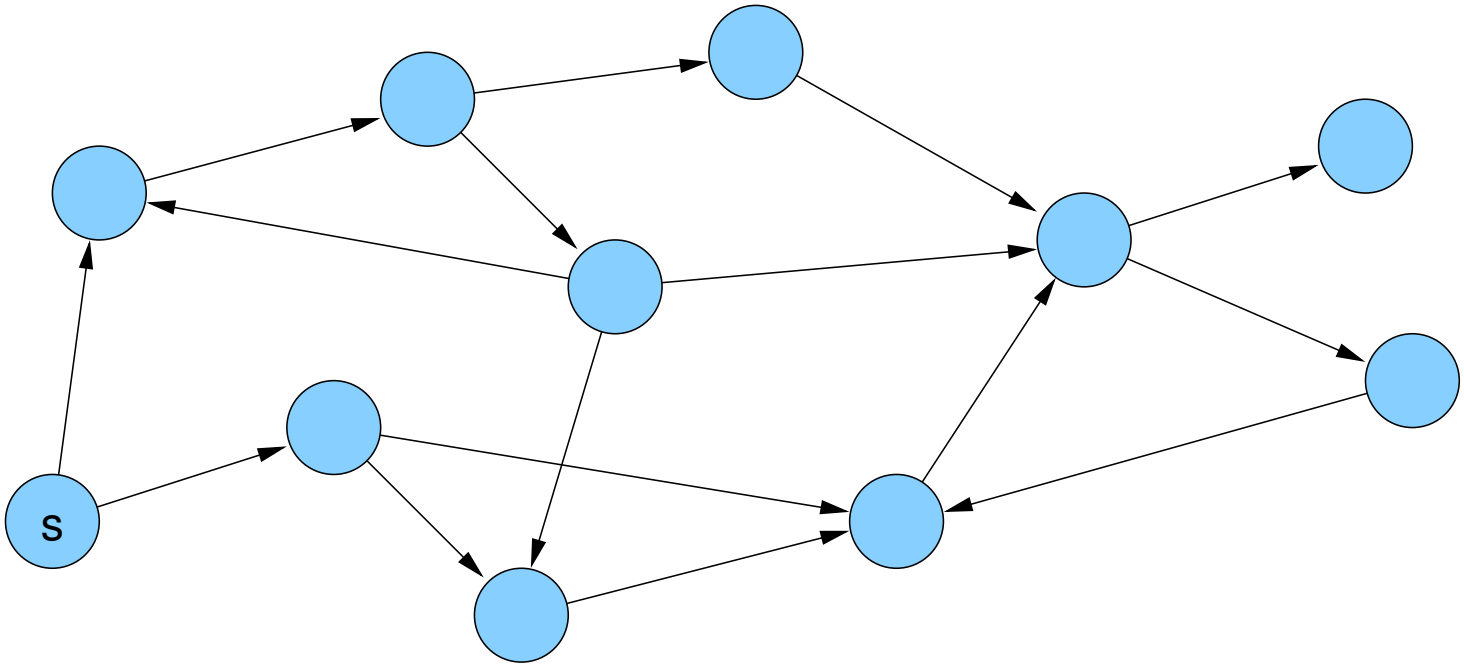
Problem:

Wie kann man die Knoten eines Graphen systematisch durchlaufen?

Grundlegende Strategien:

- Breitensuche (breadth-first search, BFS)
- Tiefensuche (depth-first search, DFS)

Breitensuche



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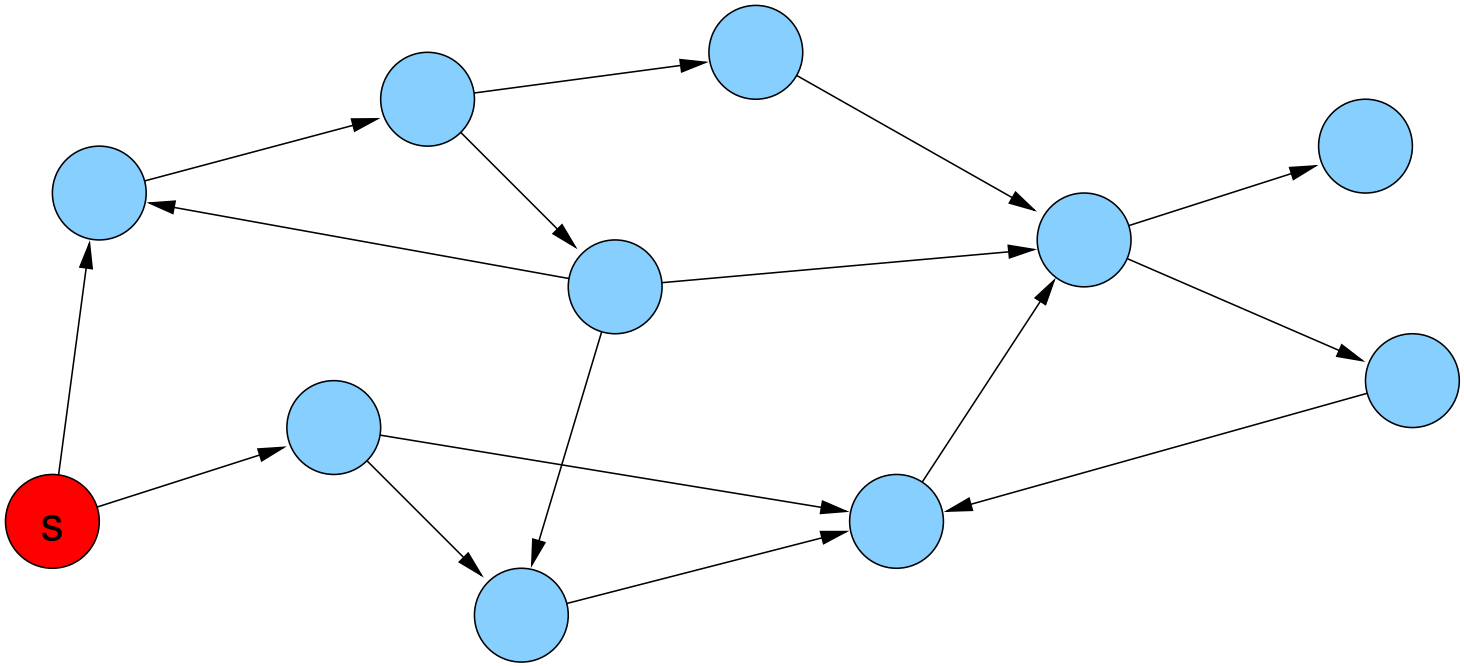

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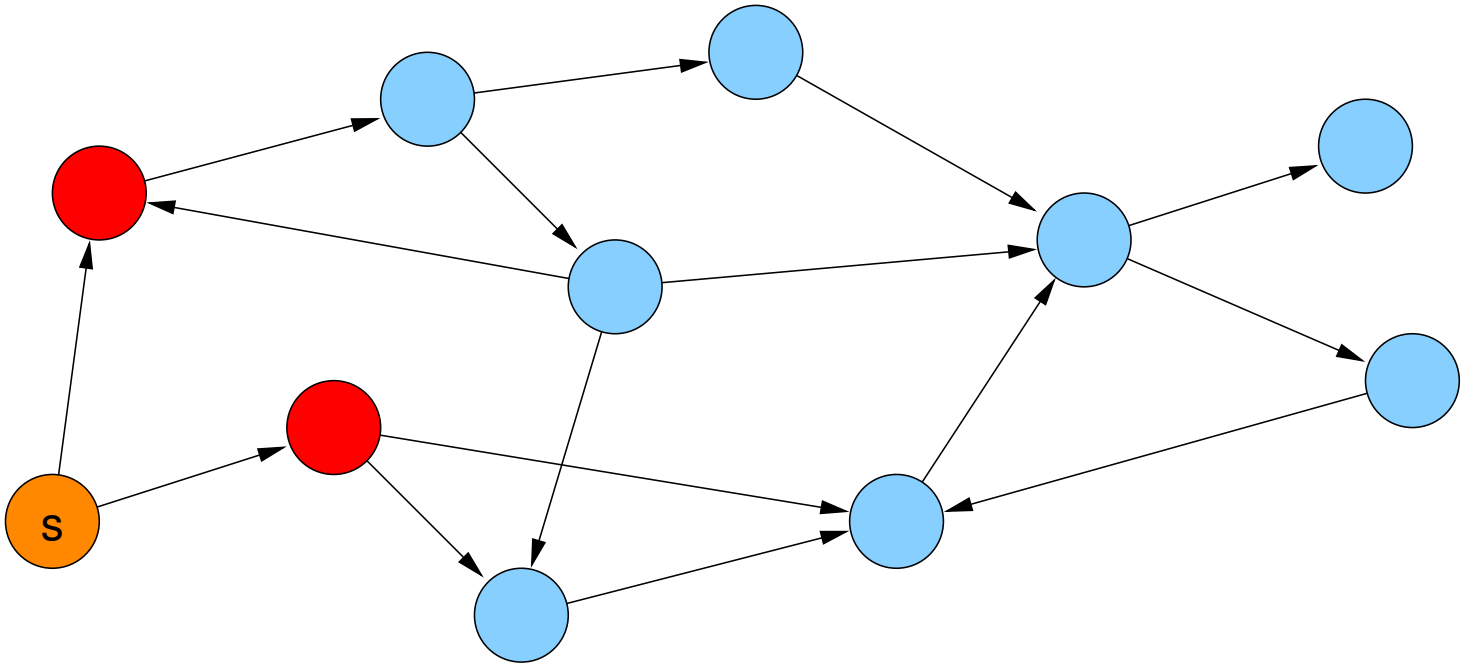

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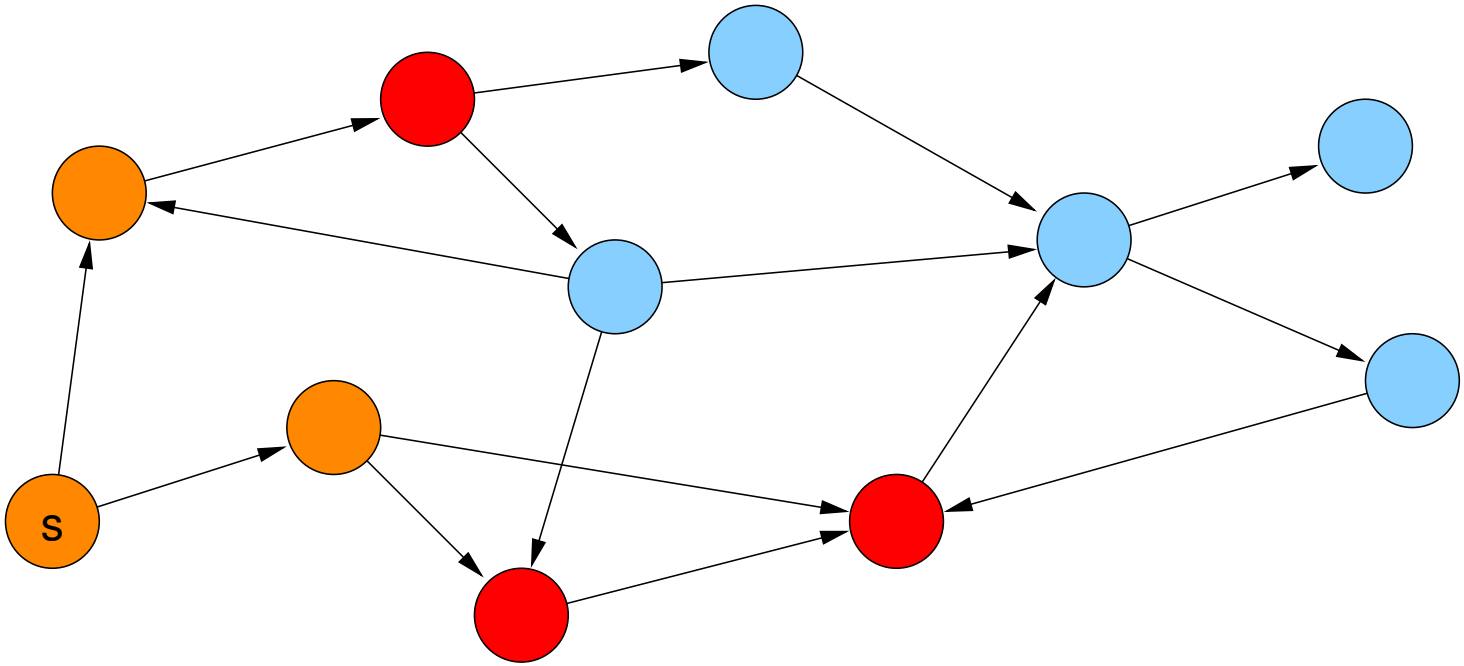

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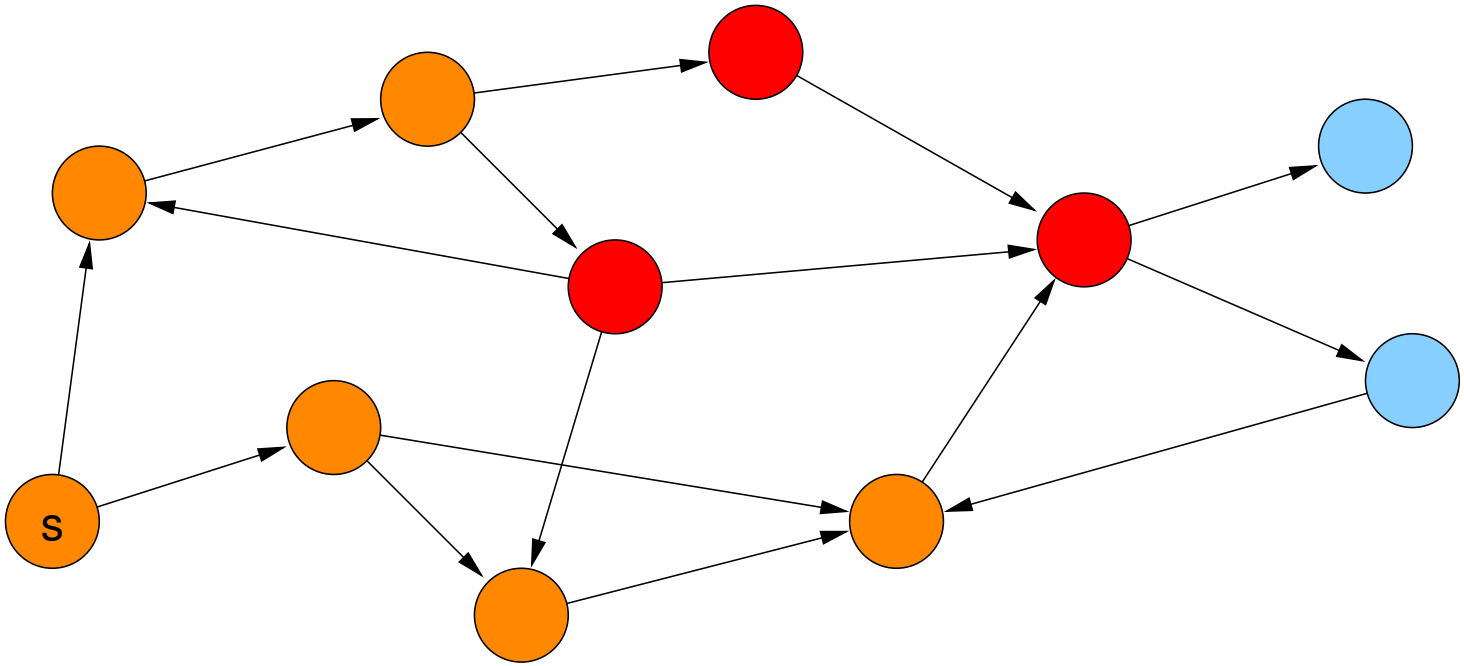

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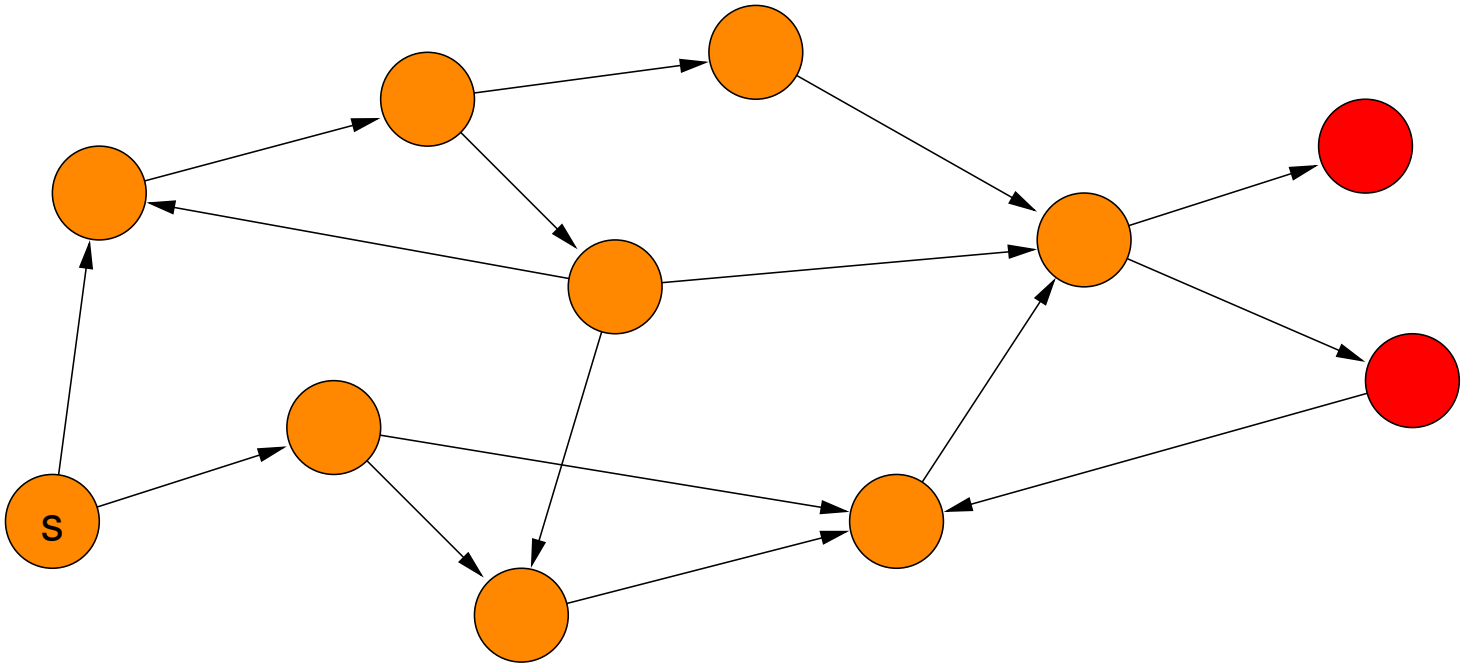

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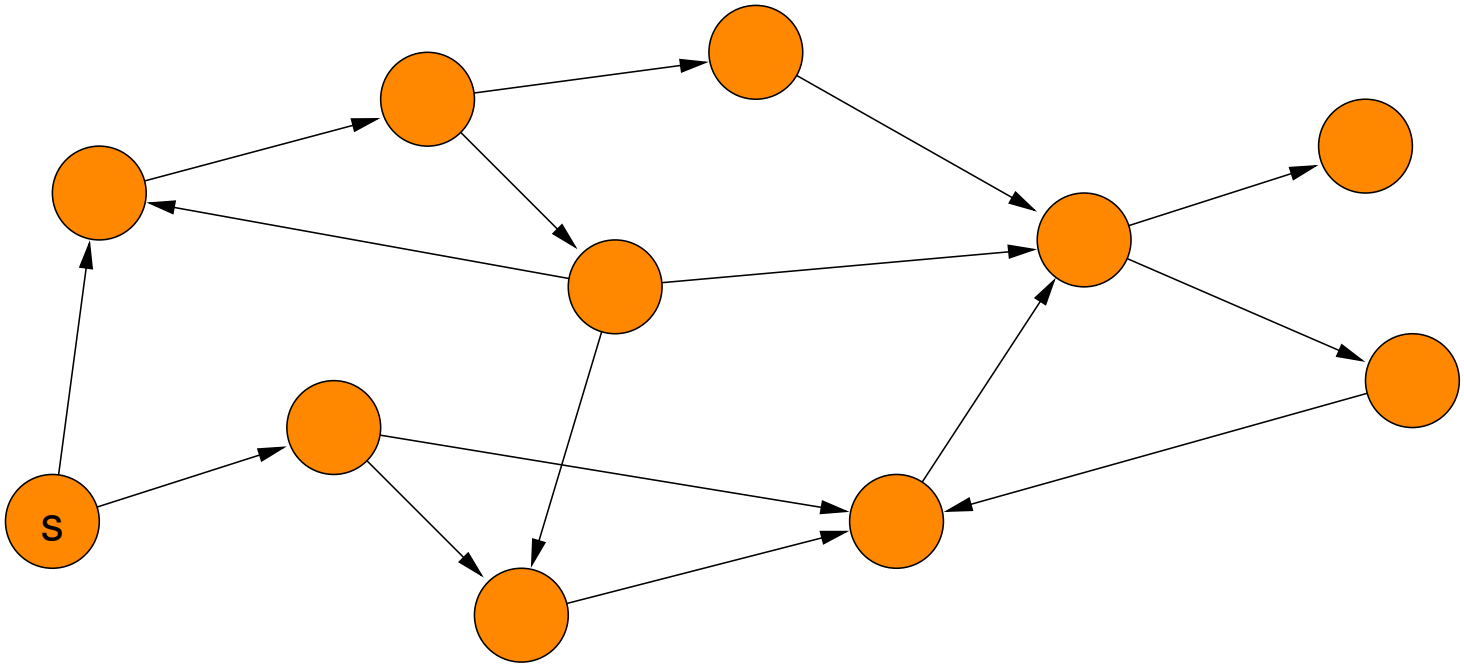

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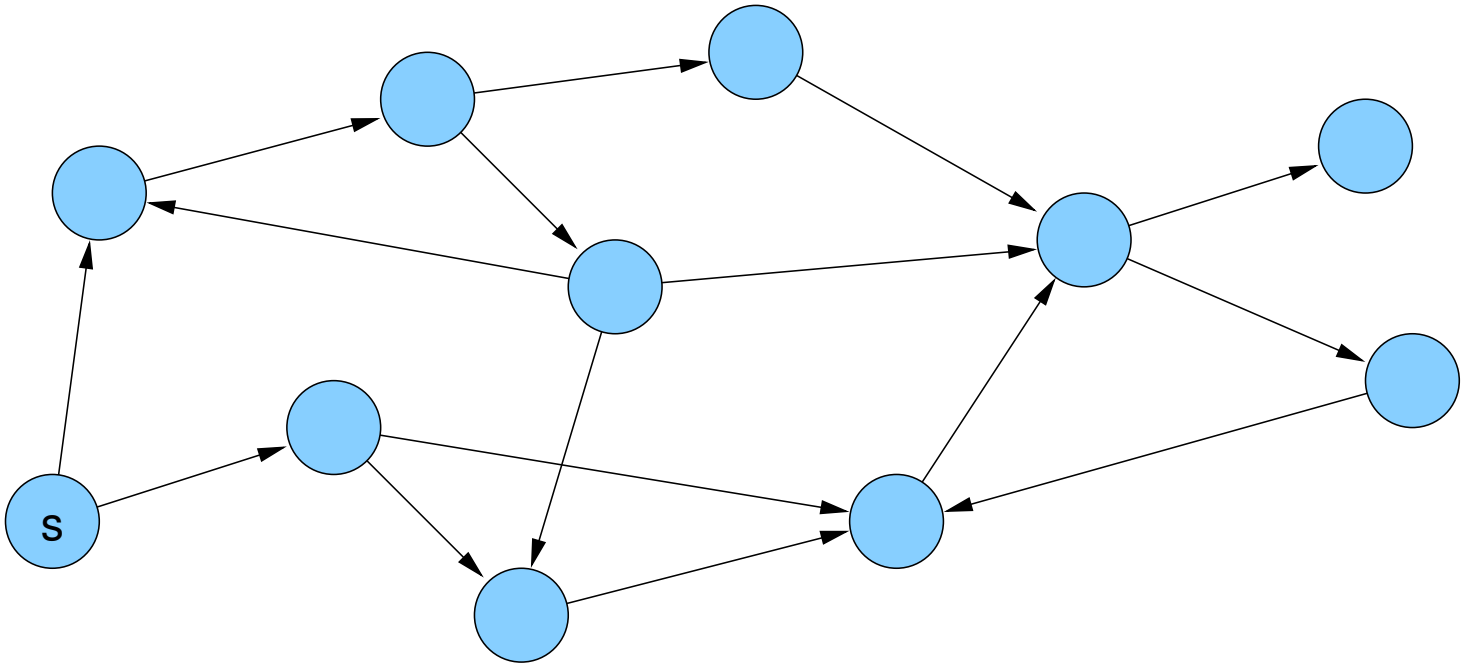

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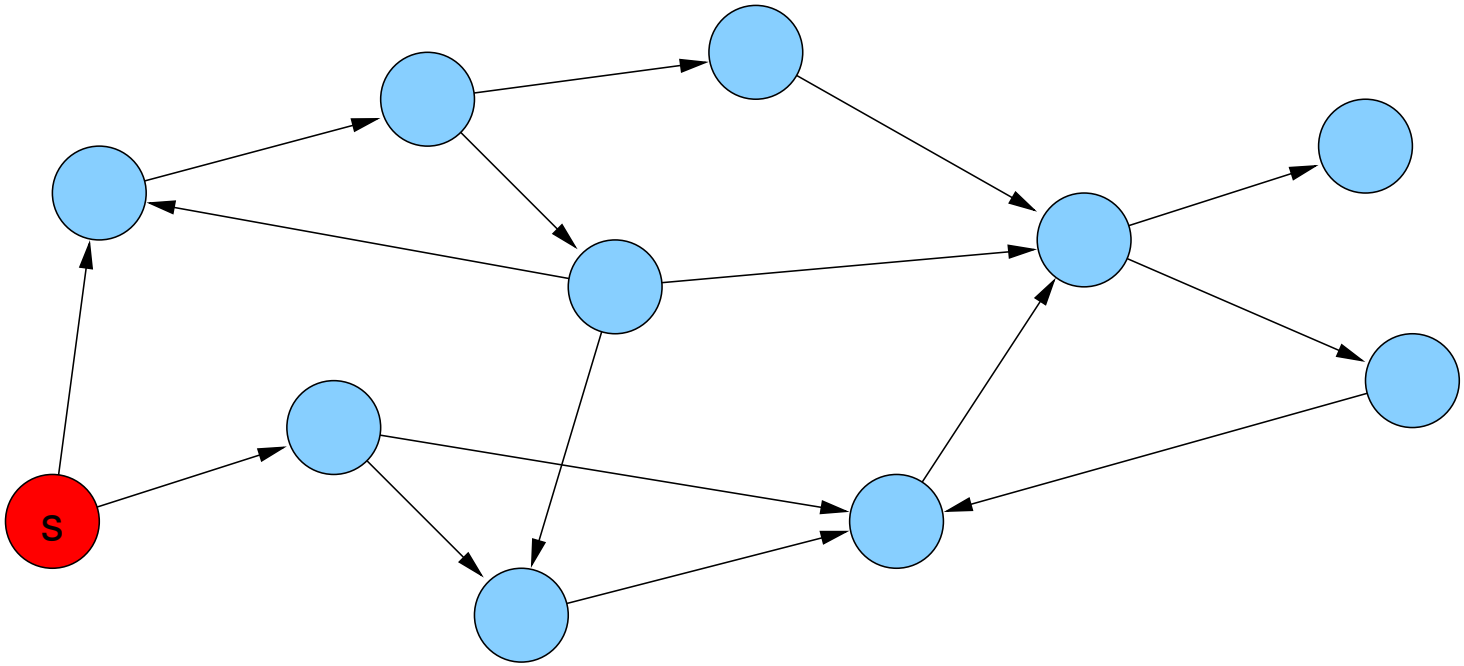

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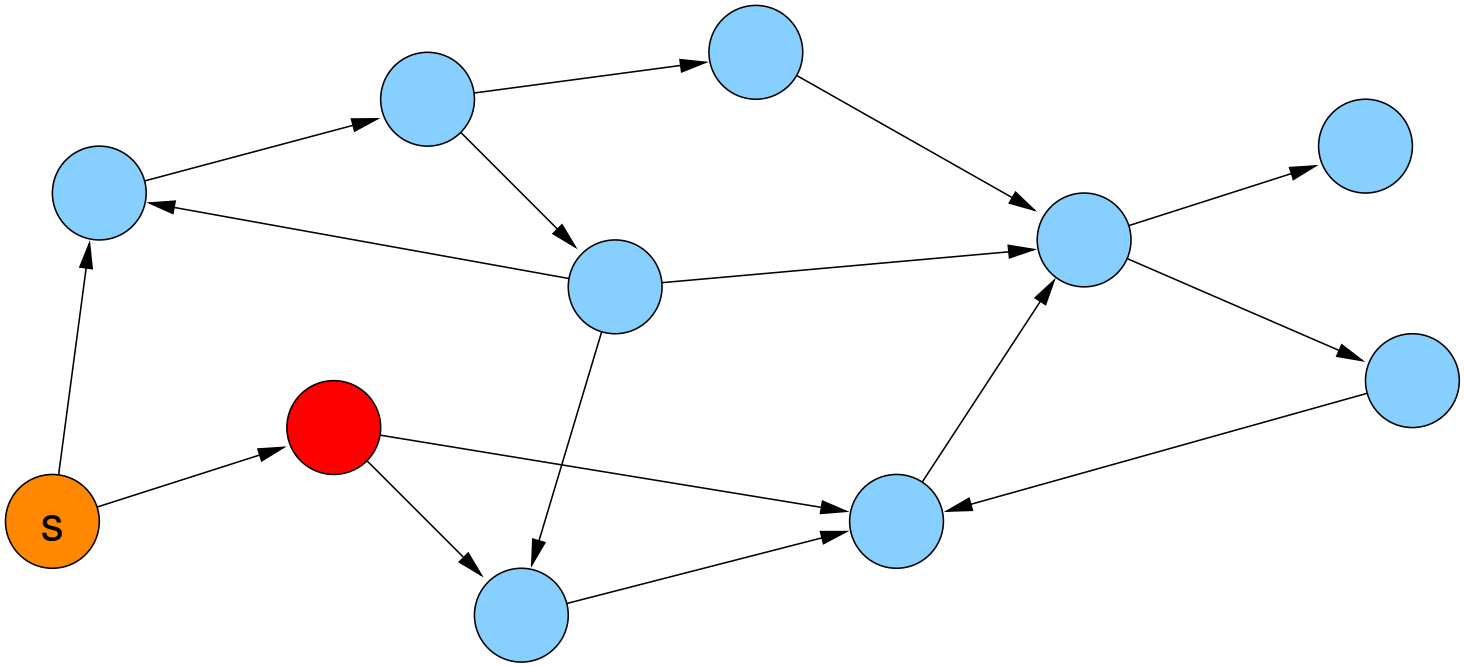

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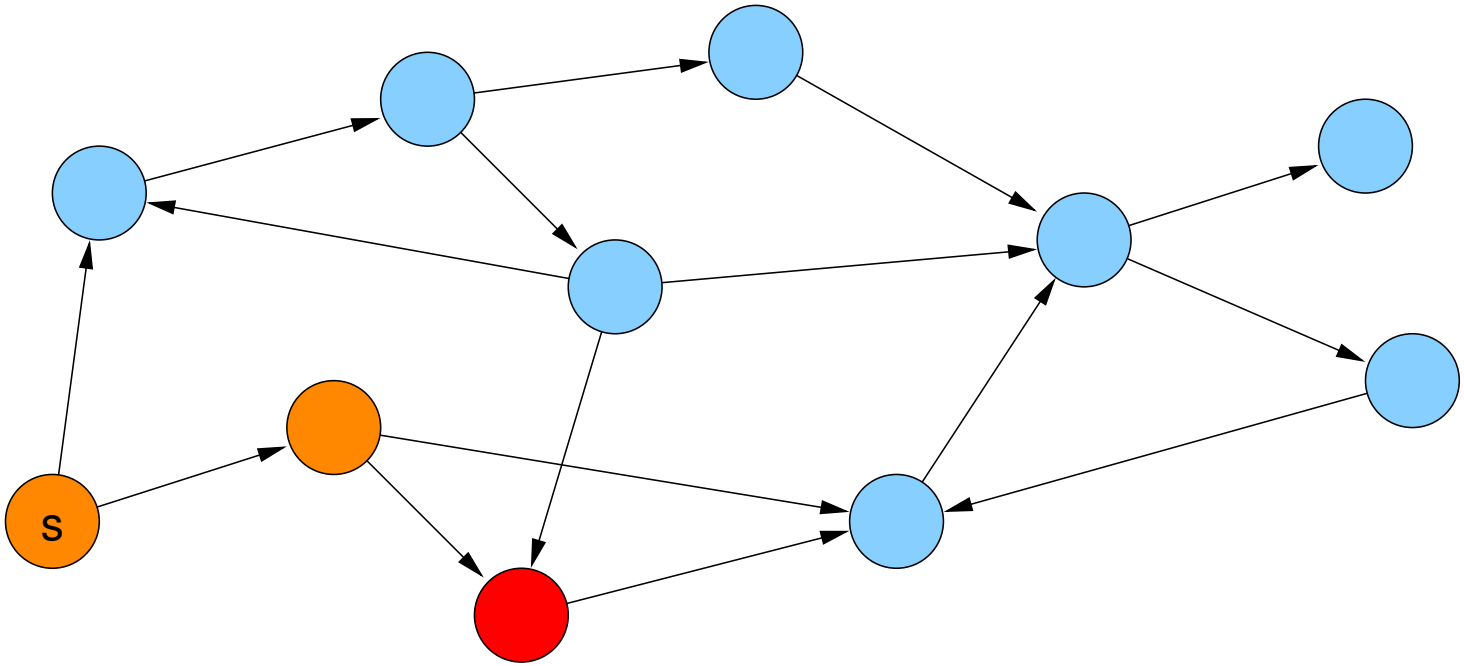

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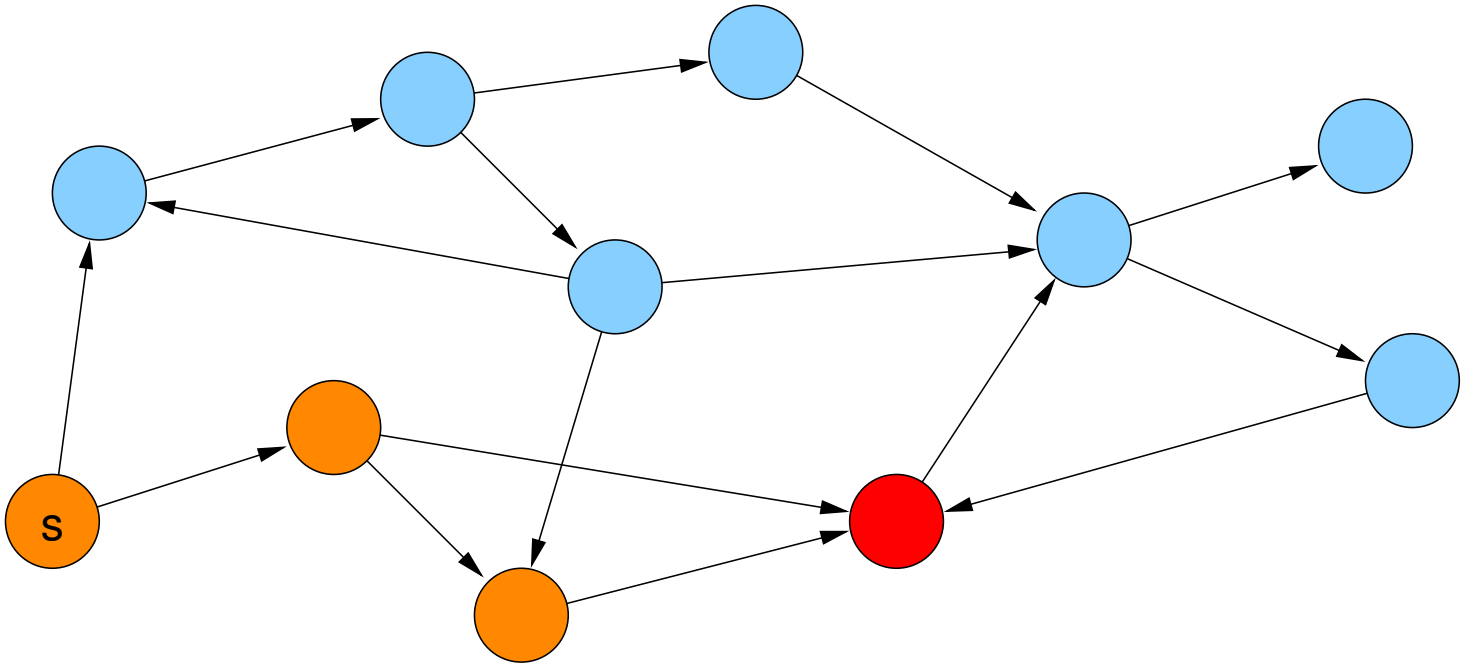

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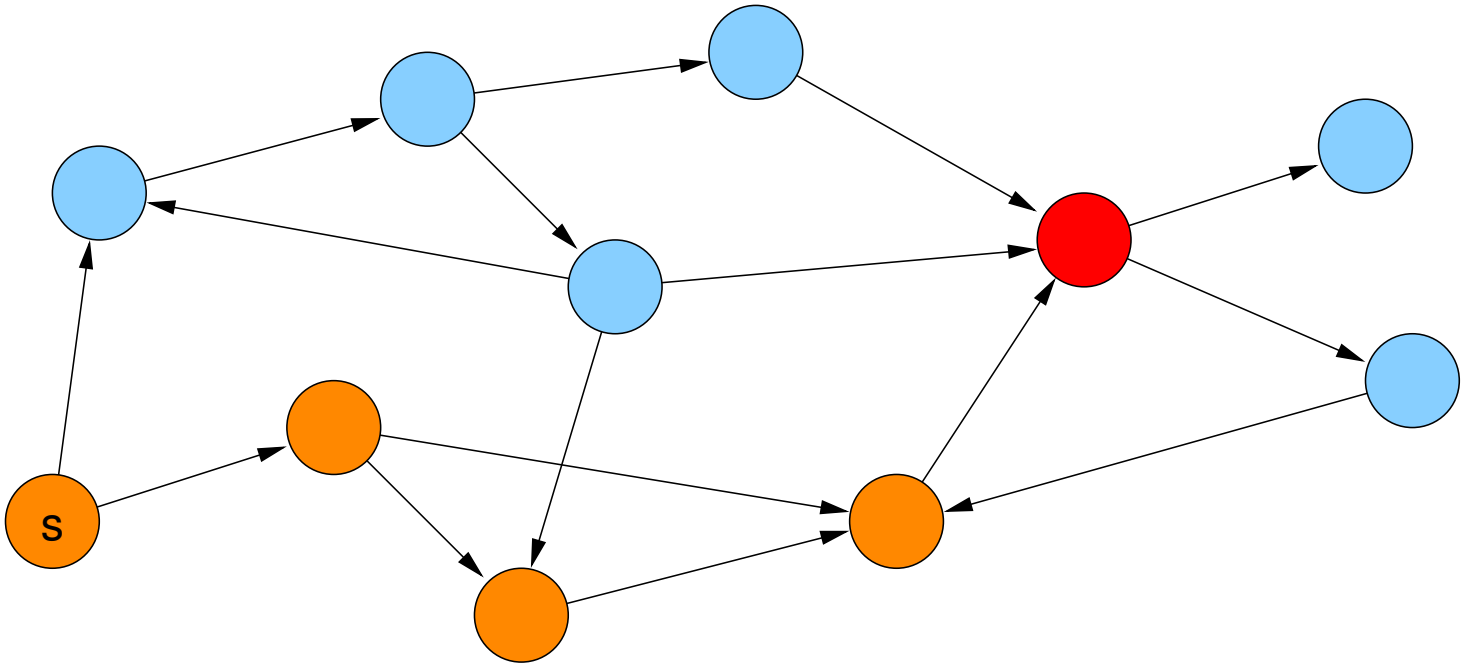

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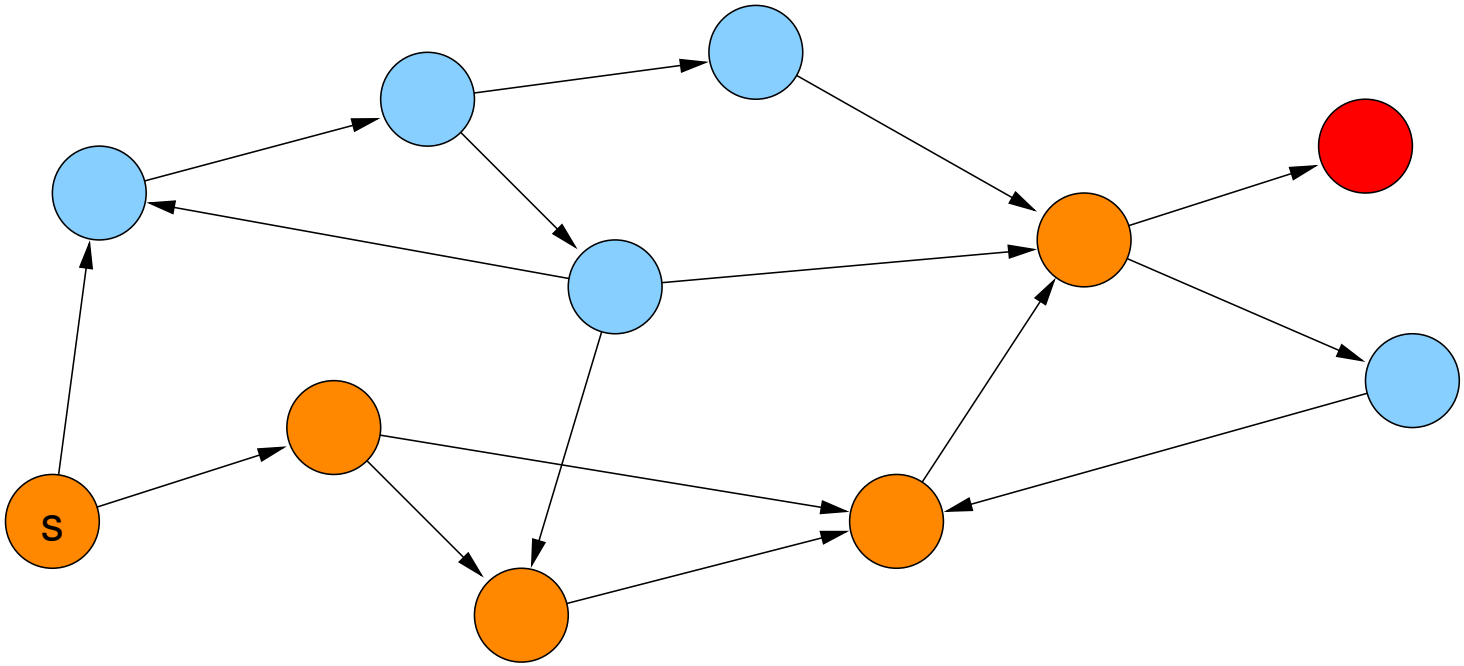

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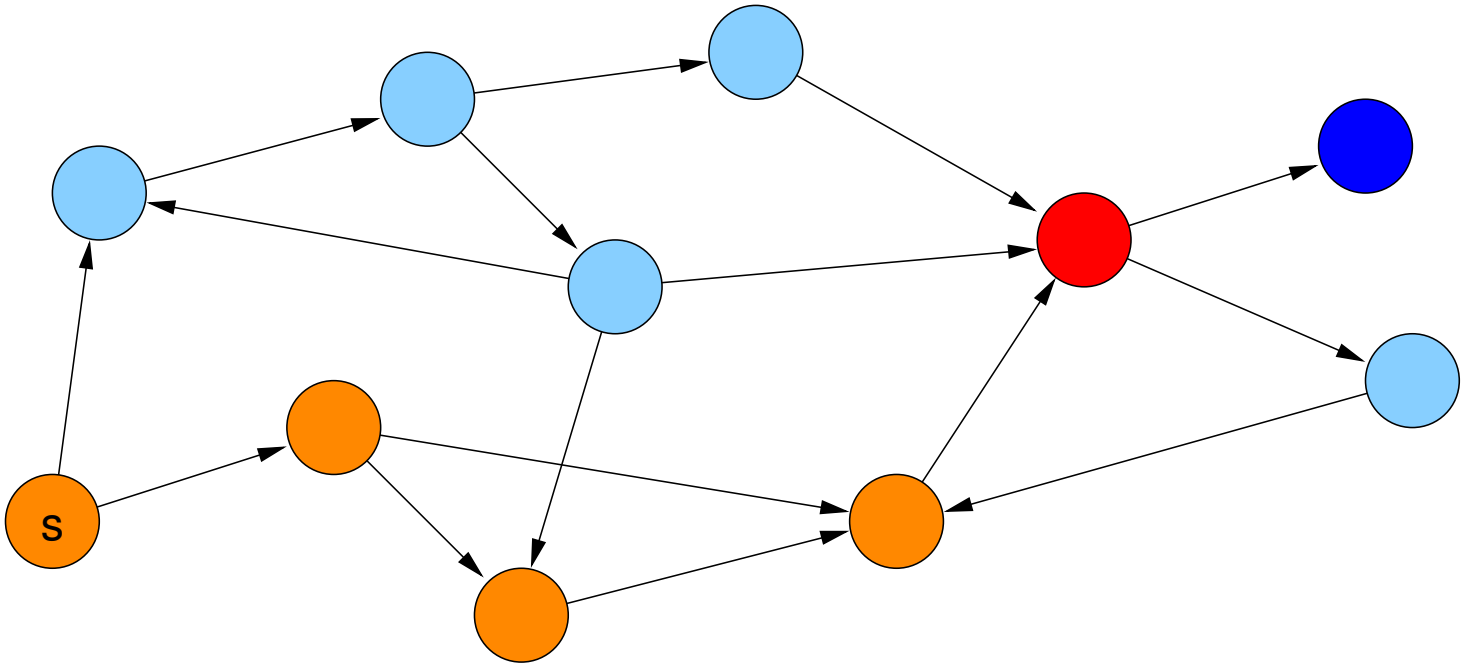

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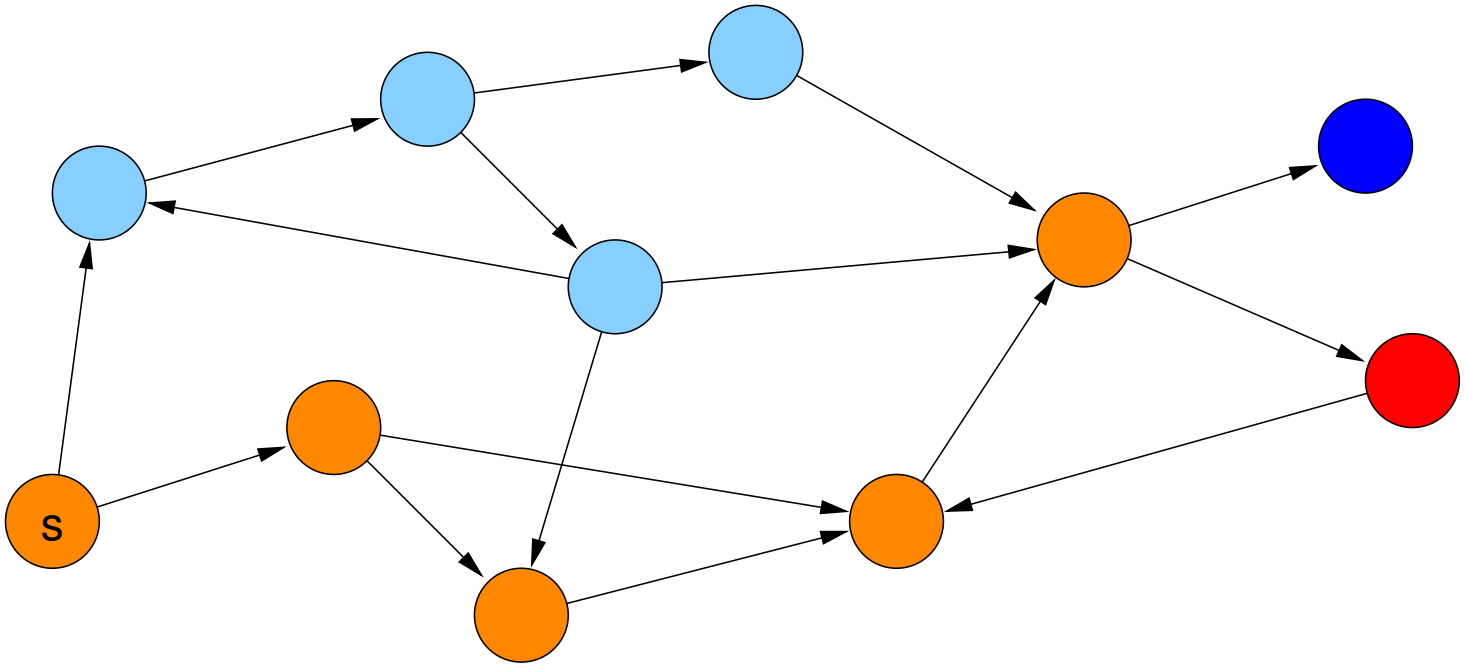

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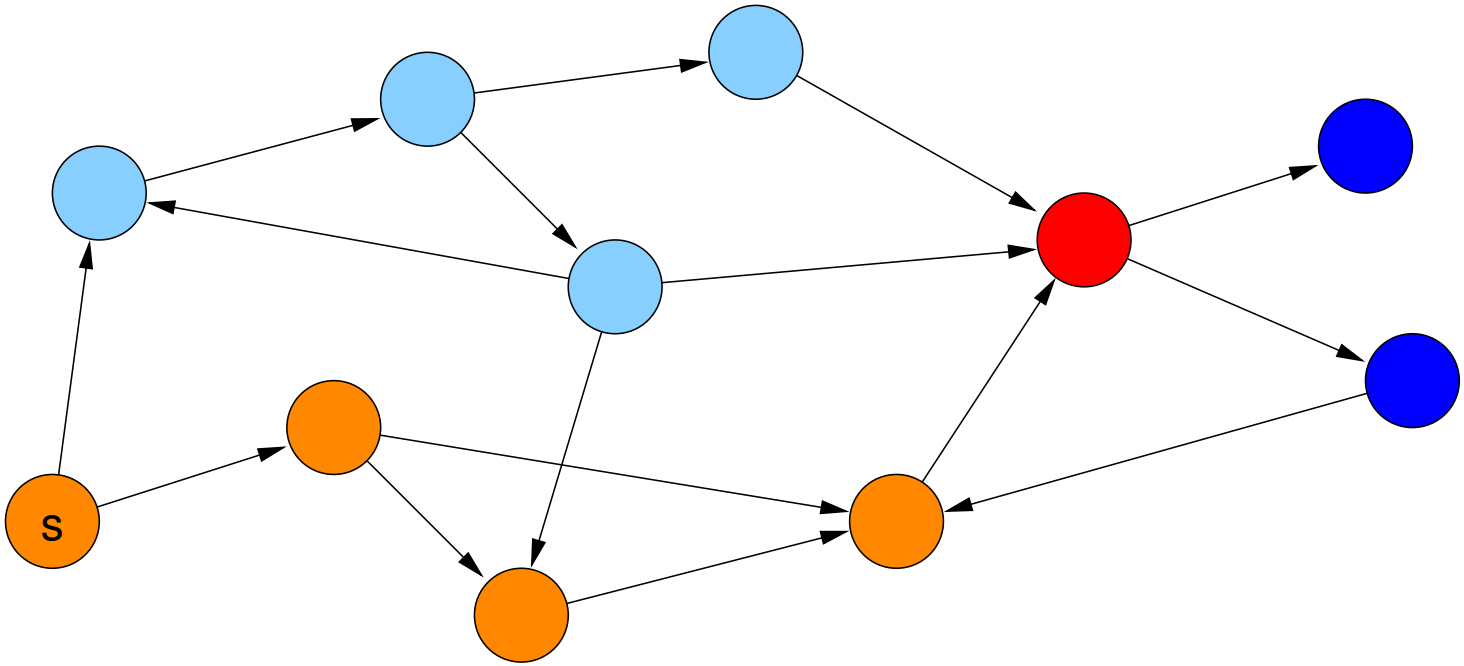

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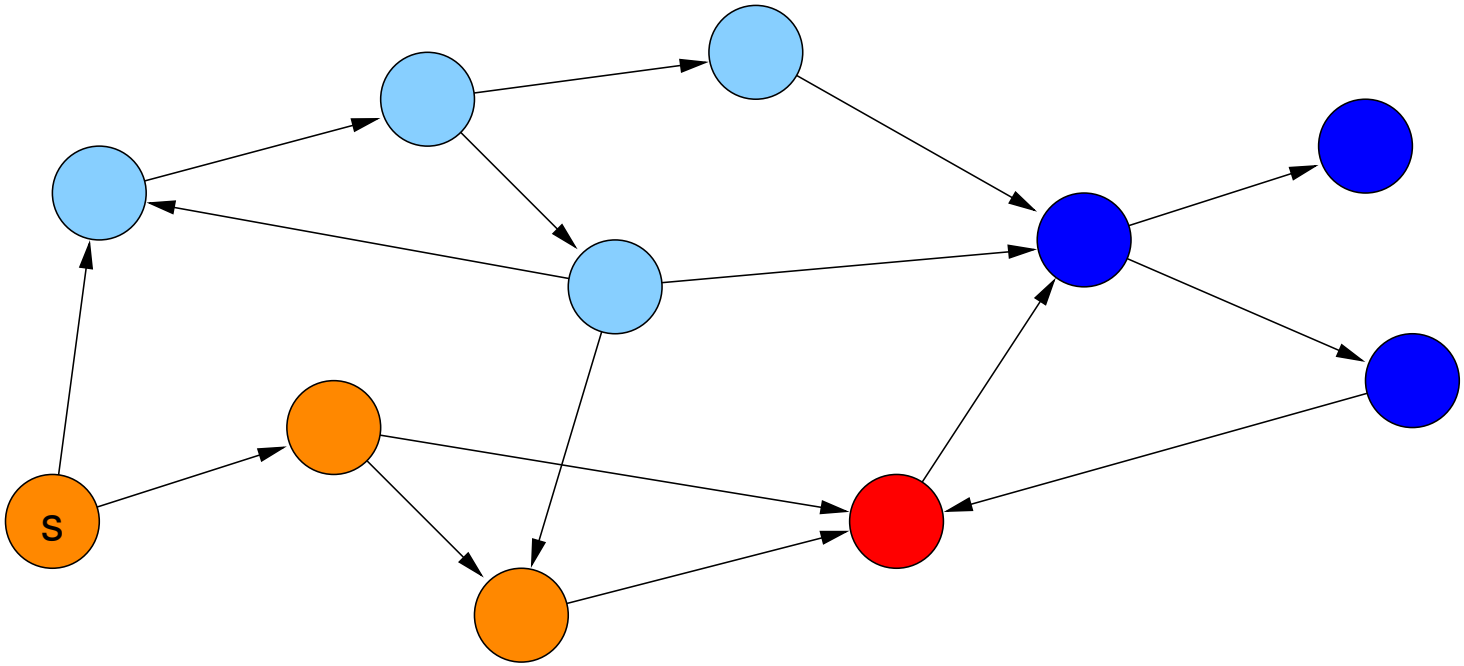

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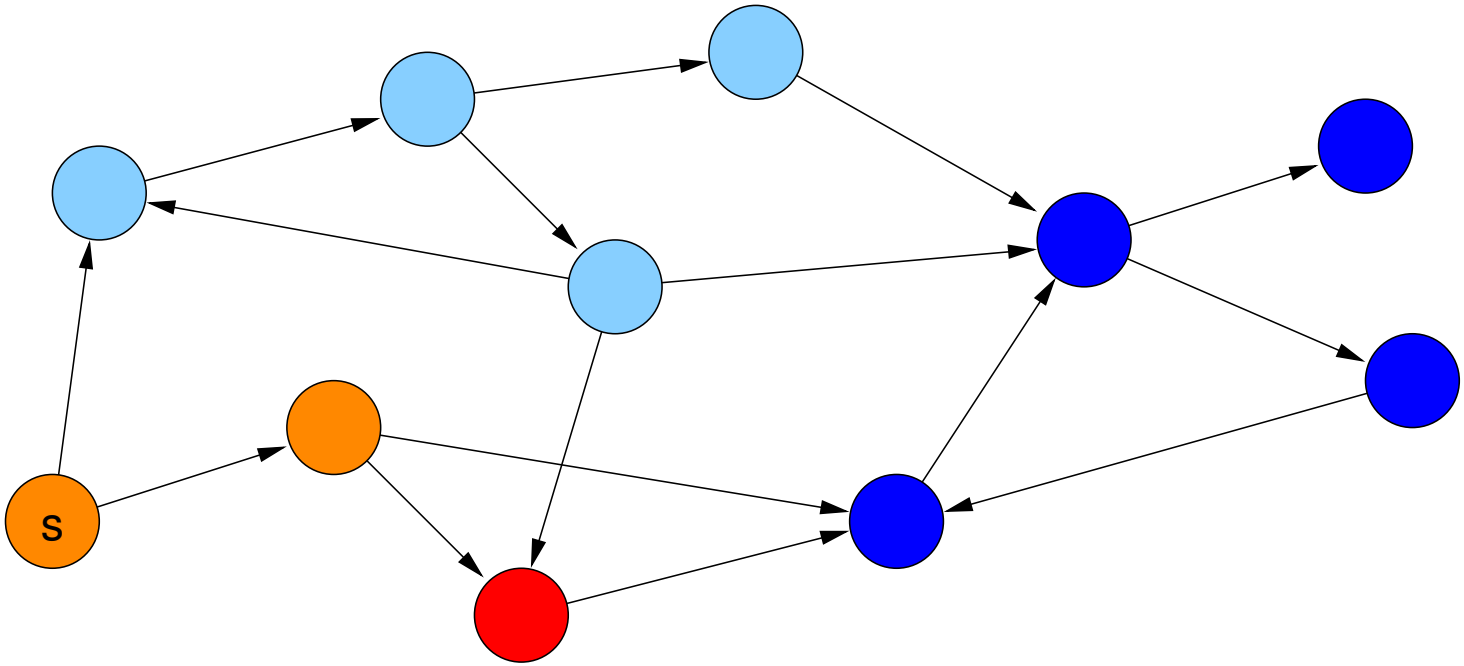

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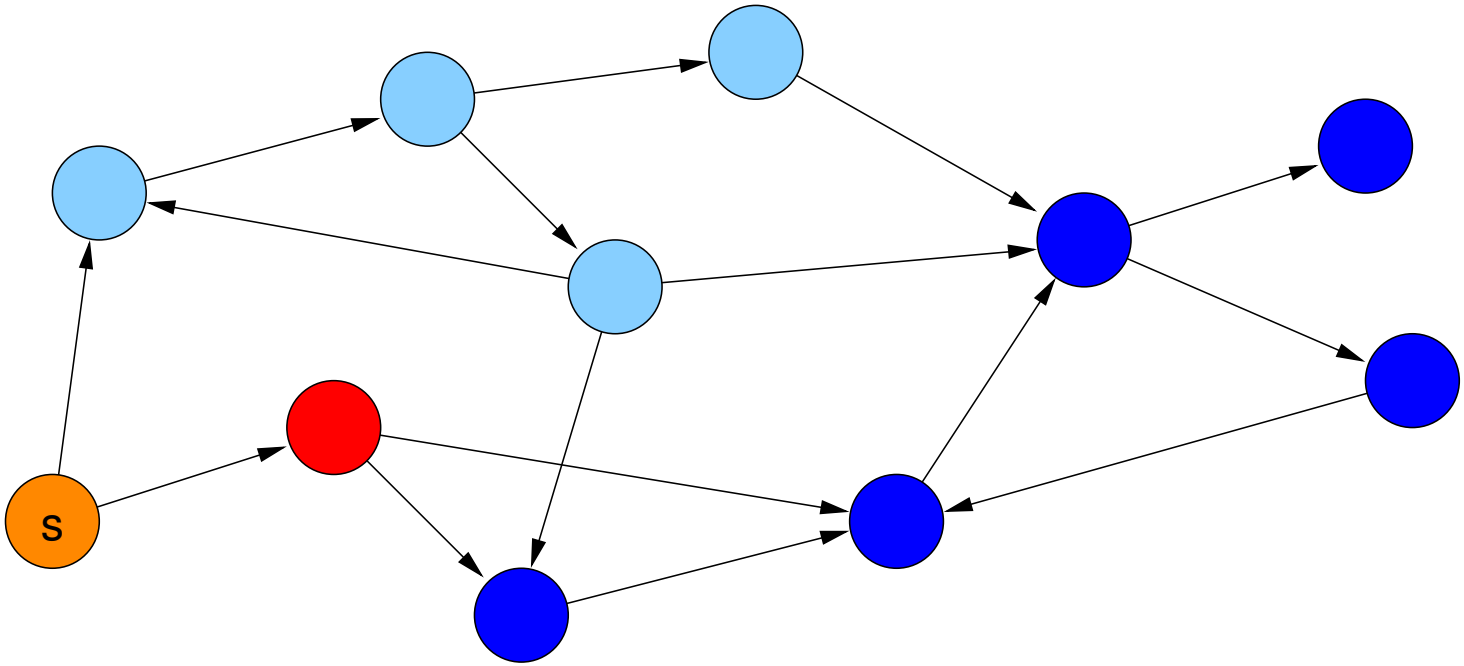

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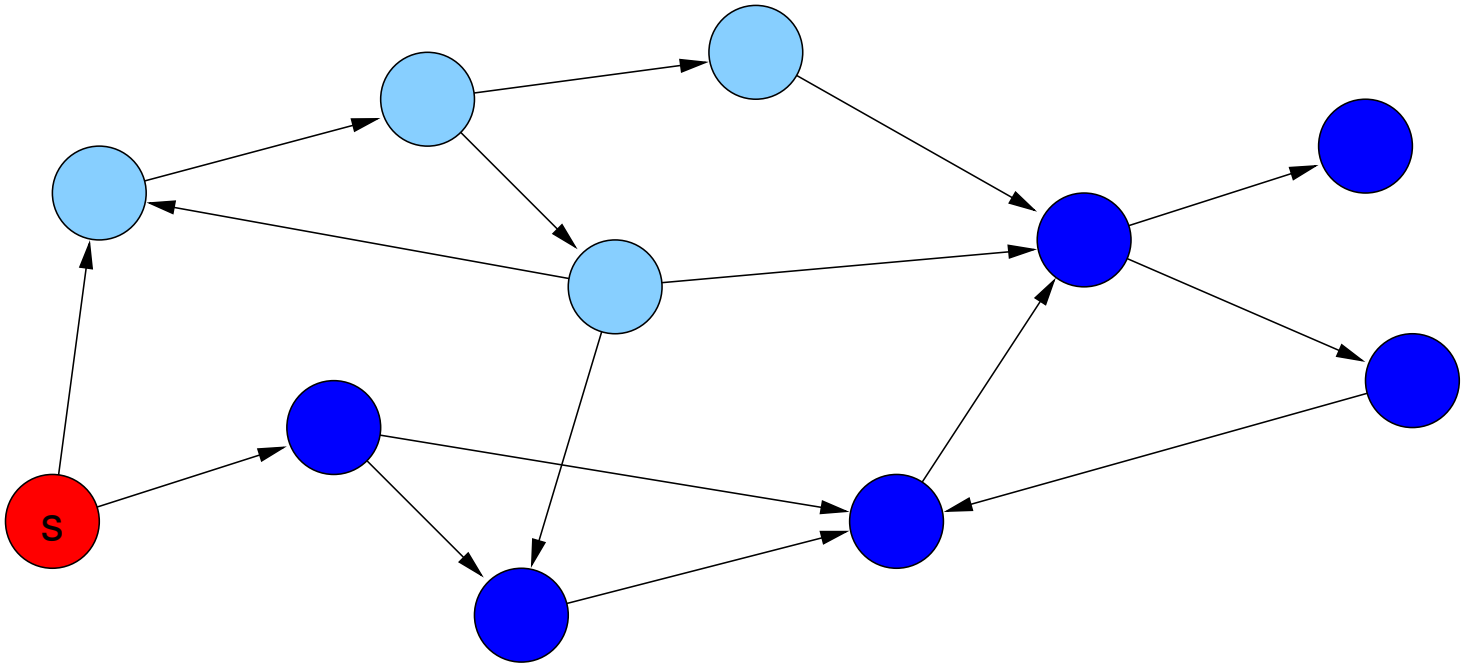

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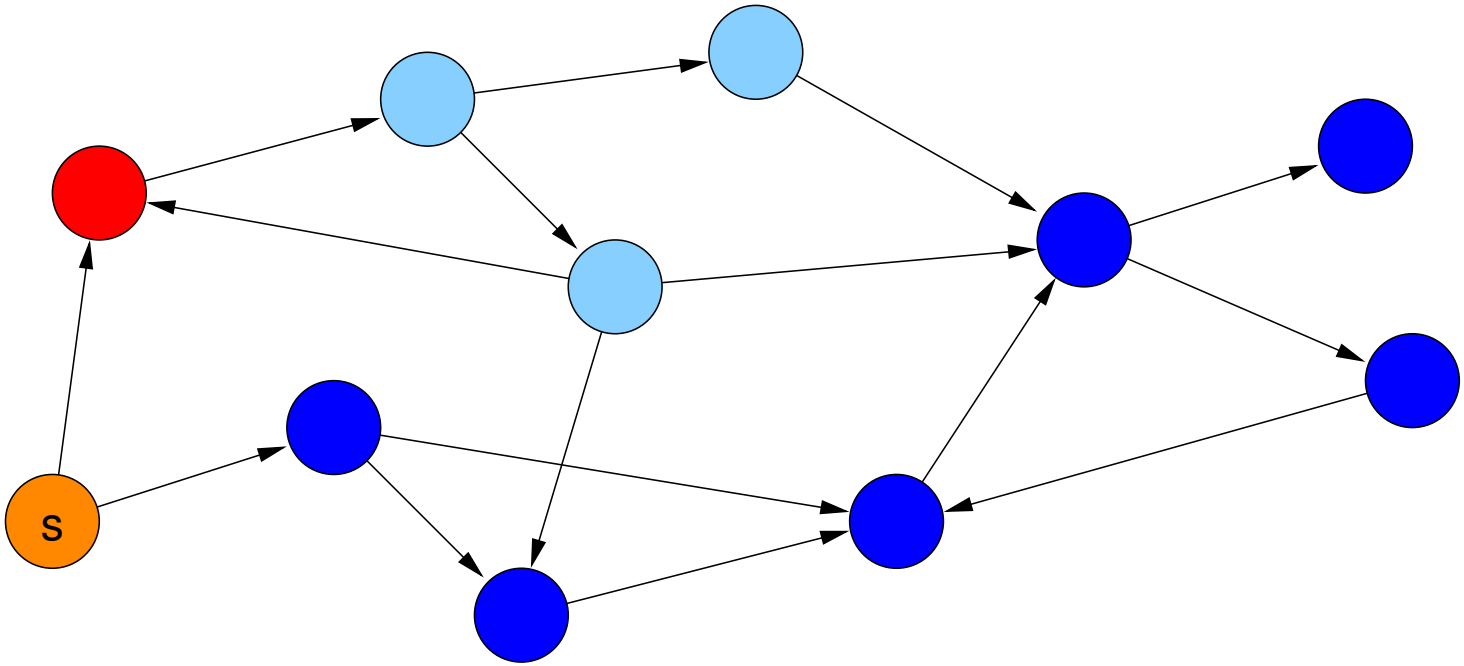

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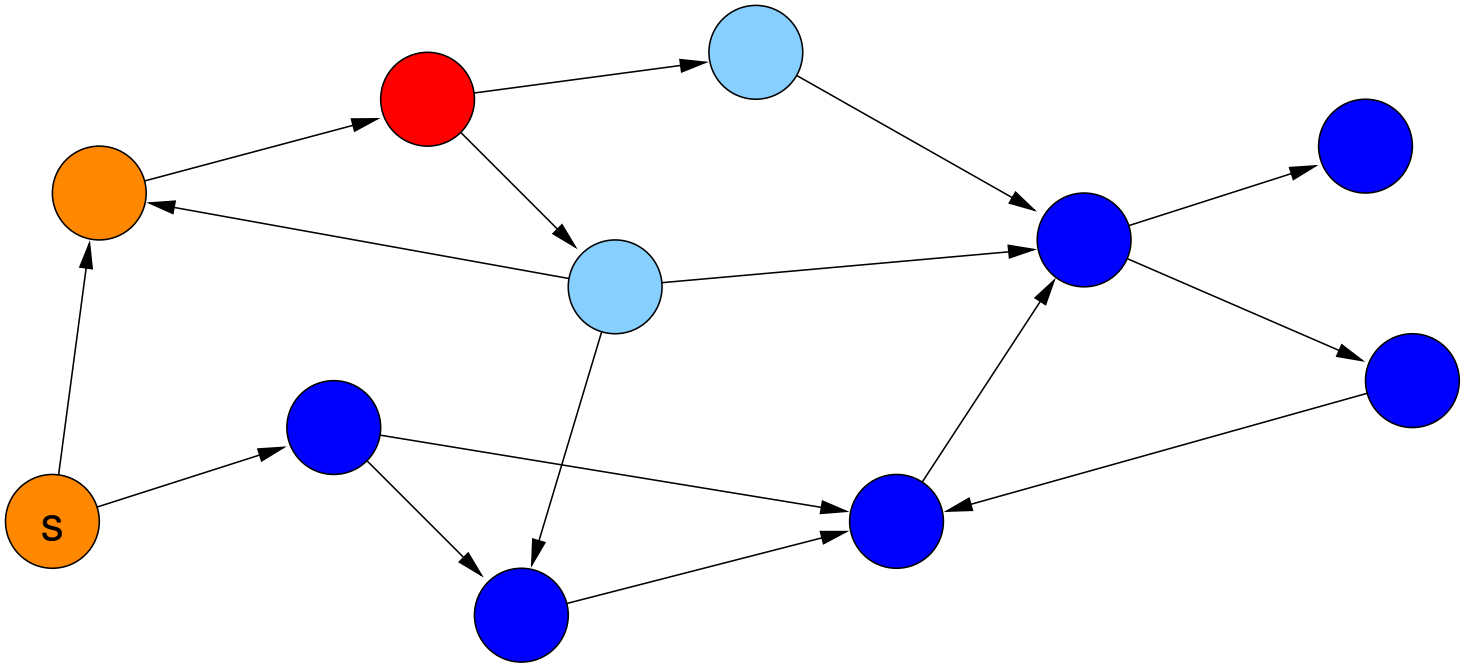

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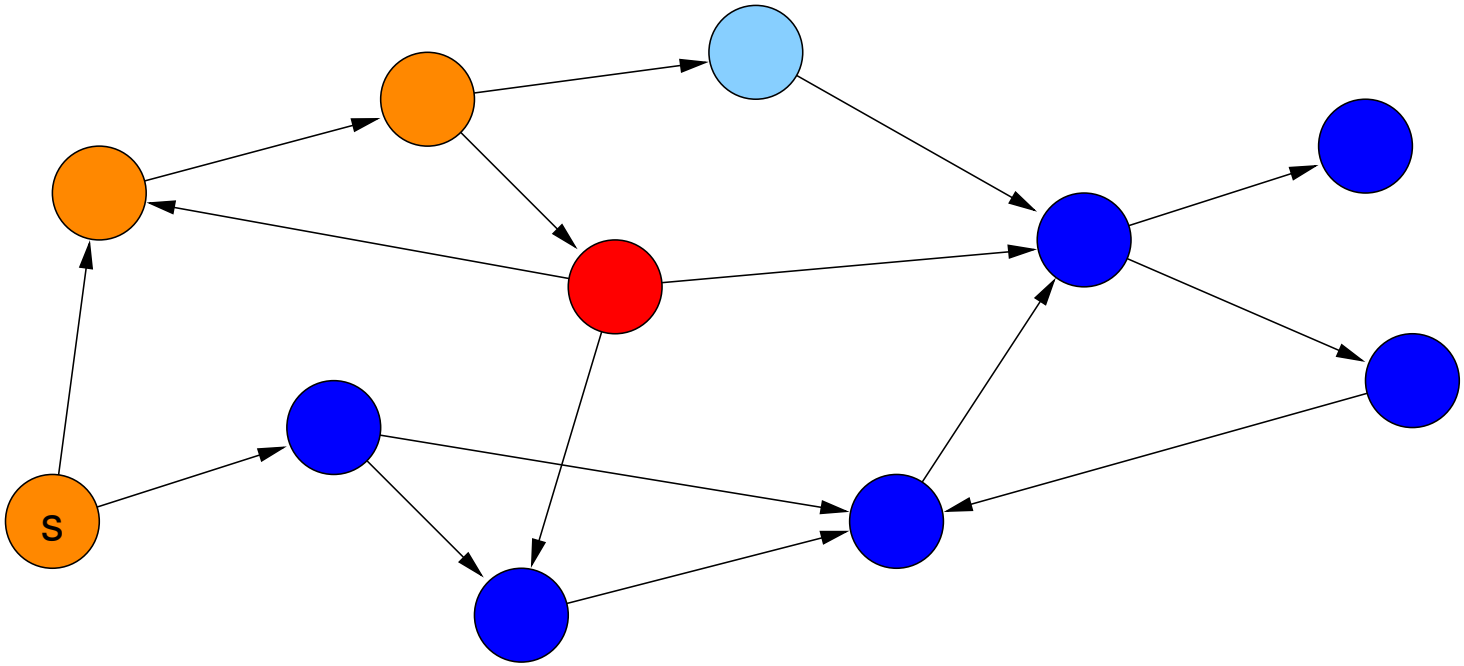

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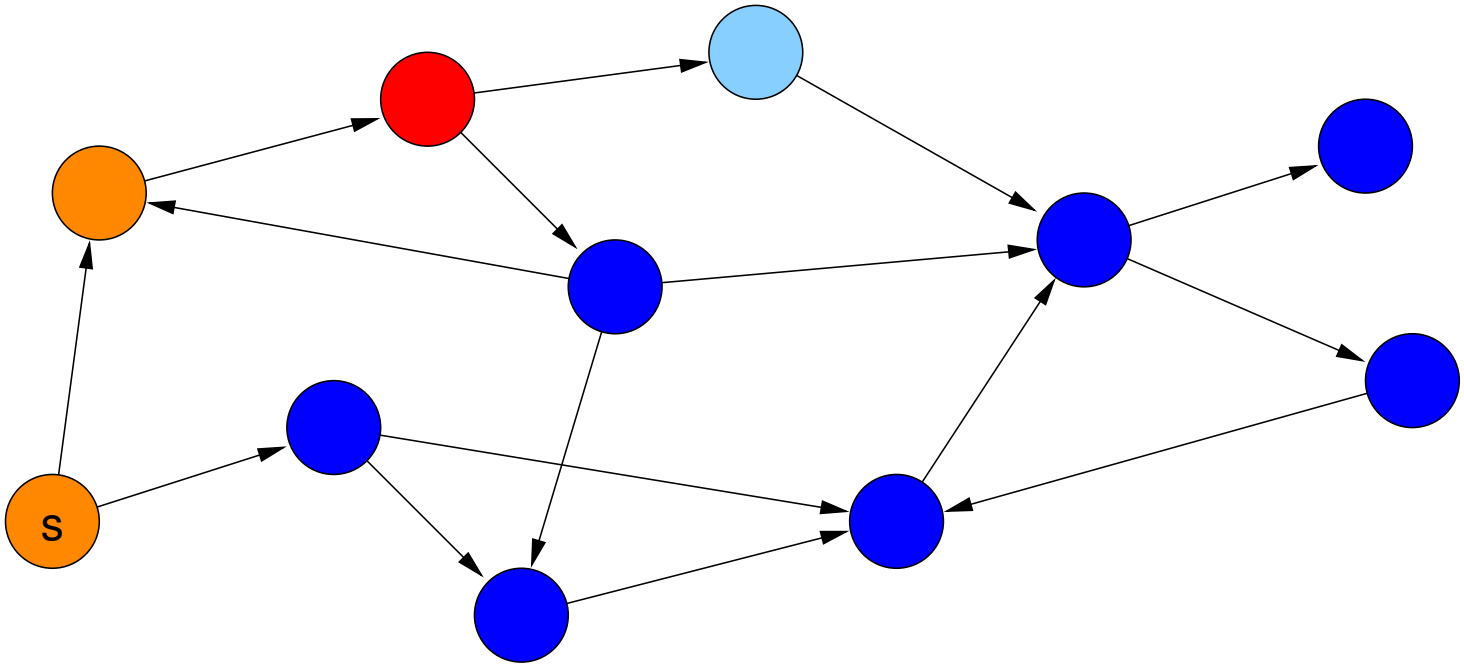

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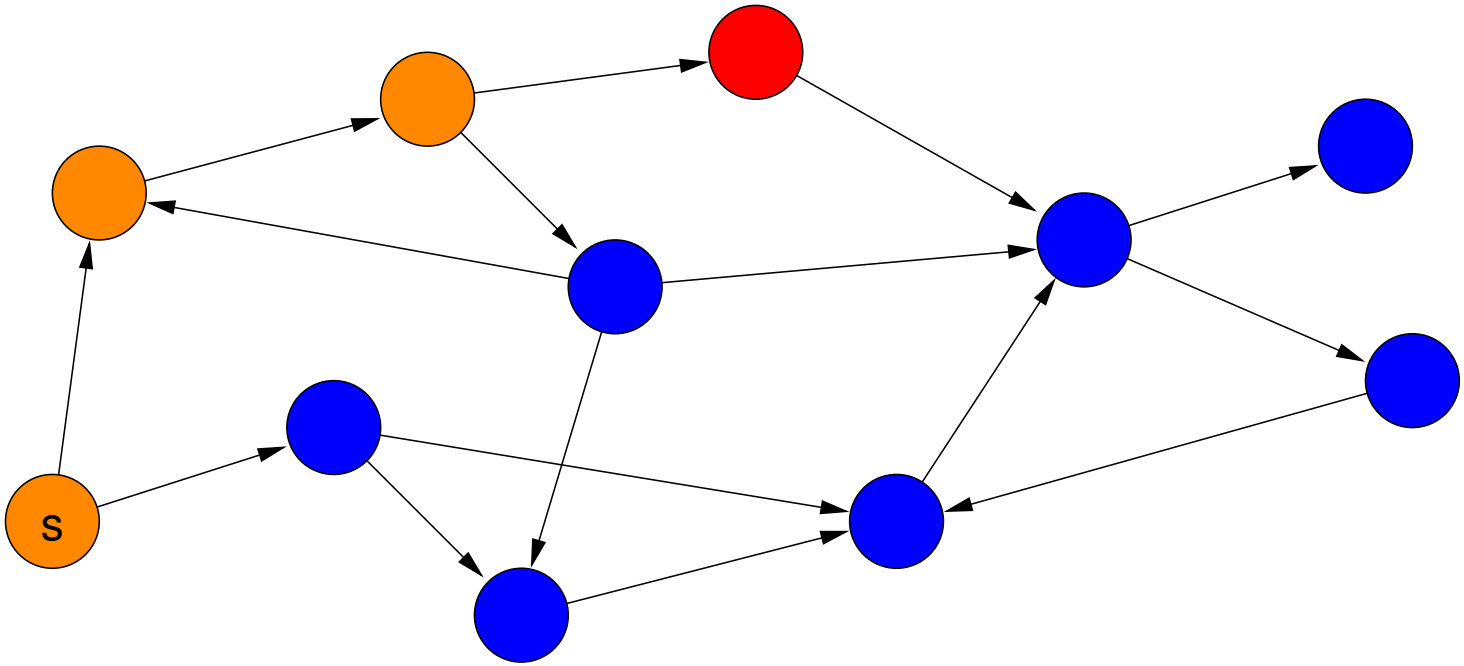

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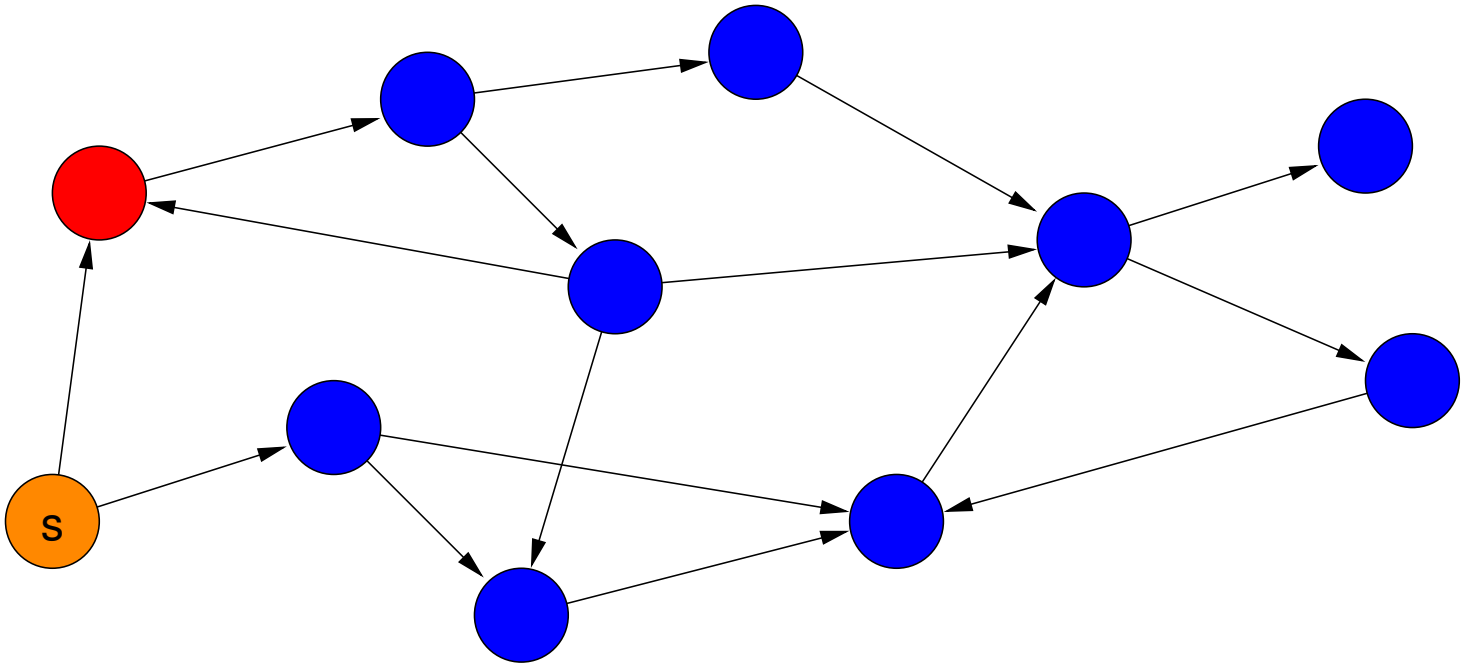

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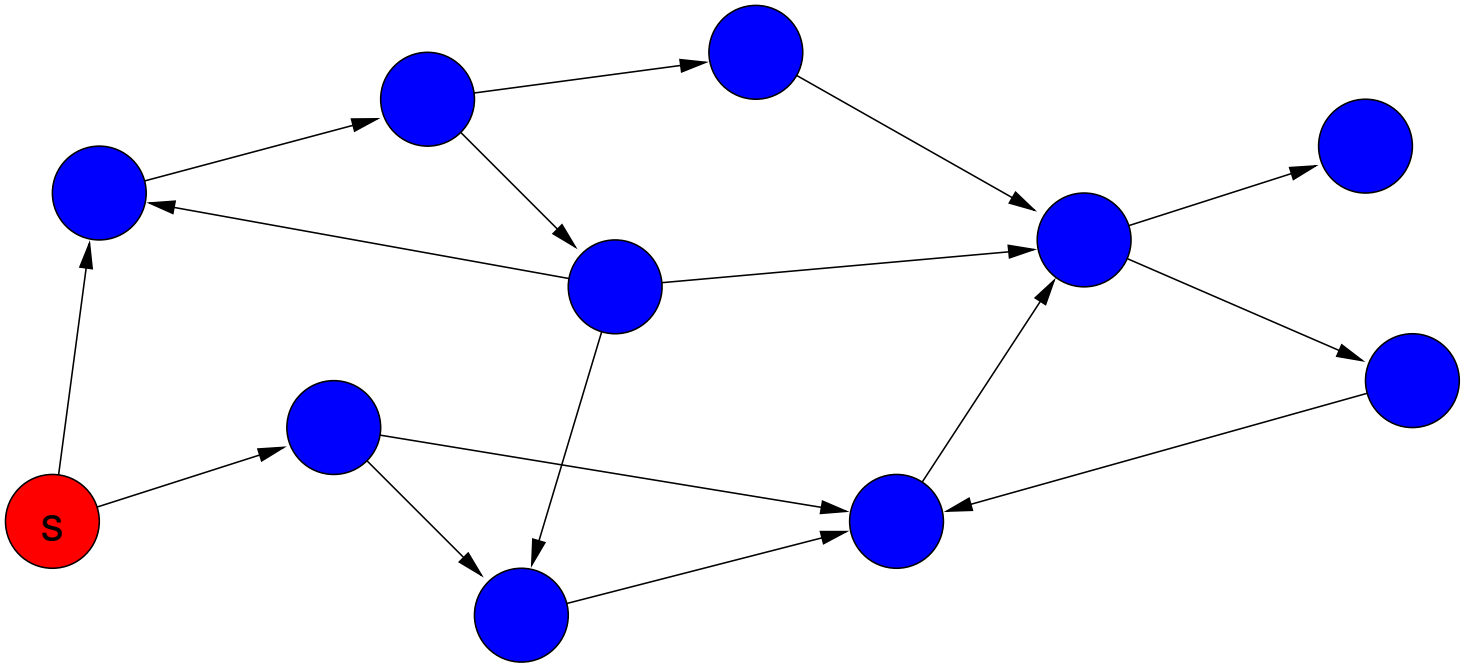

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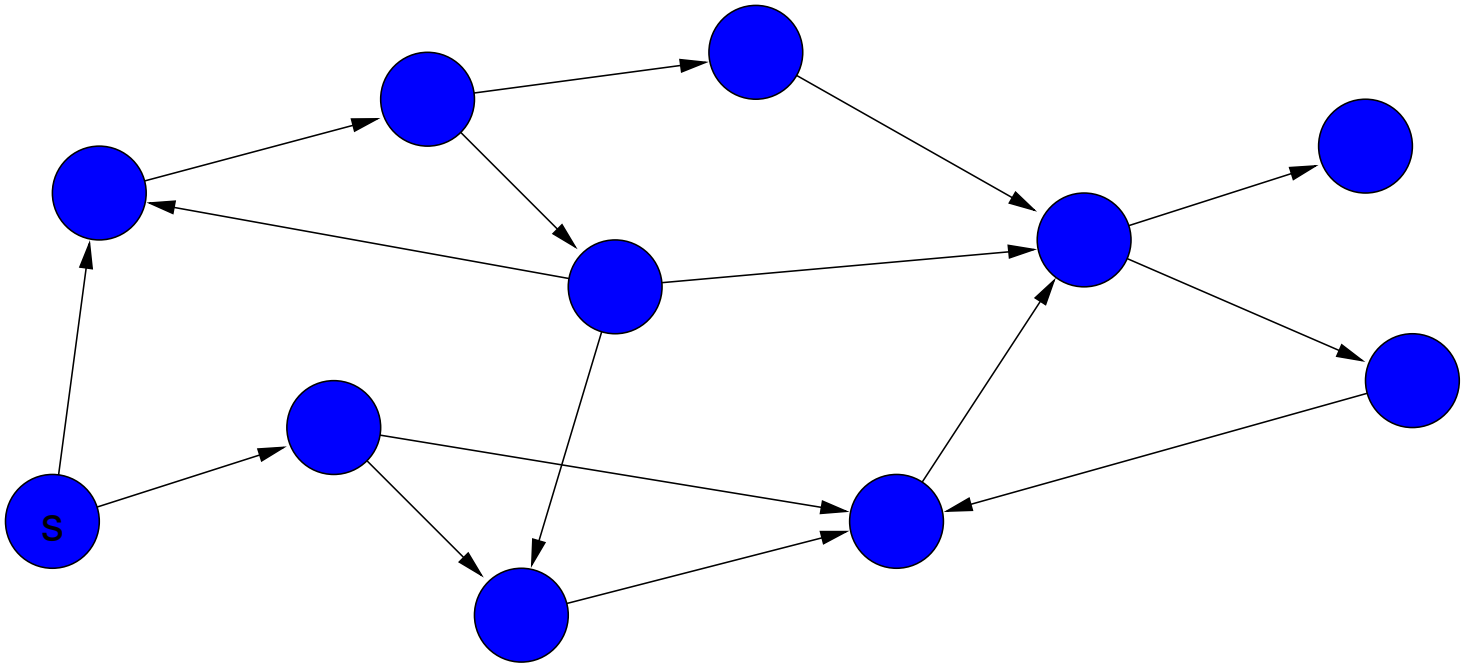
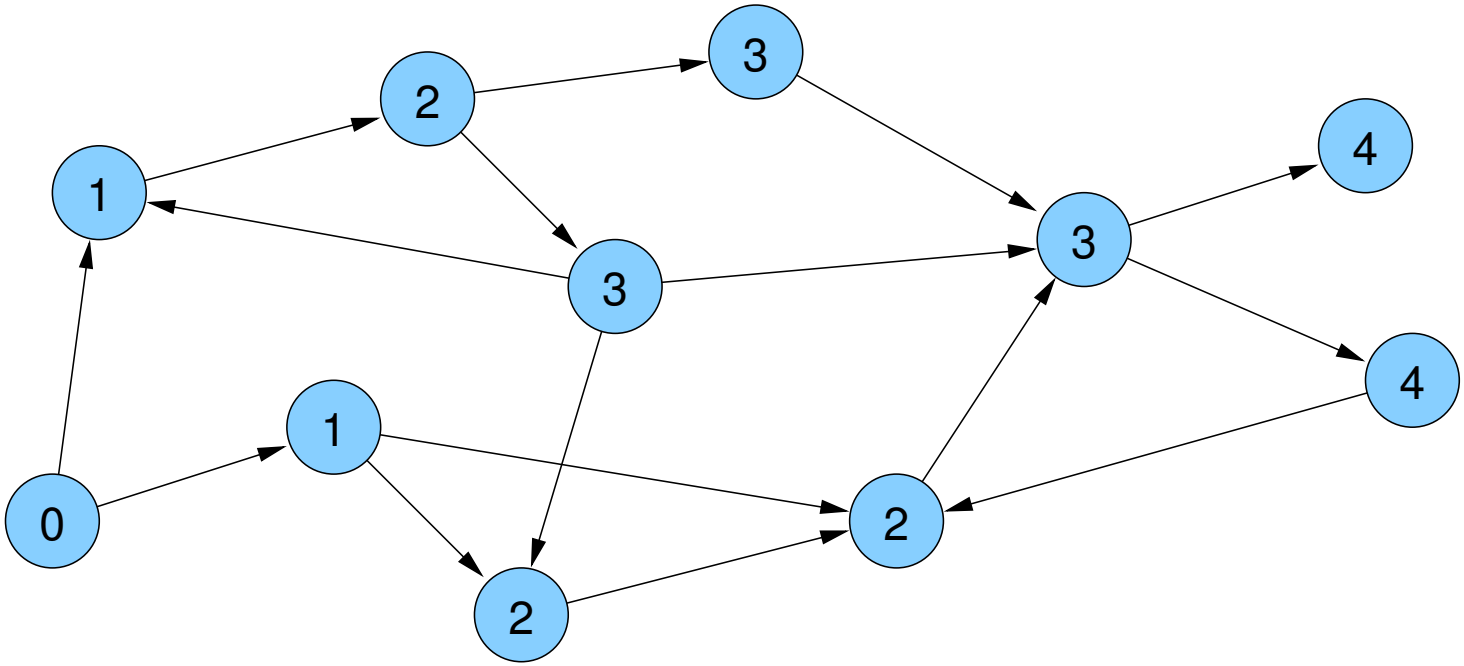


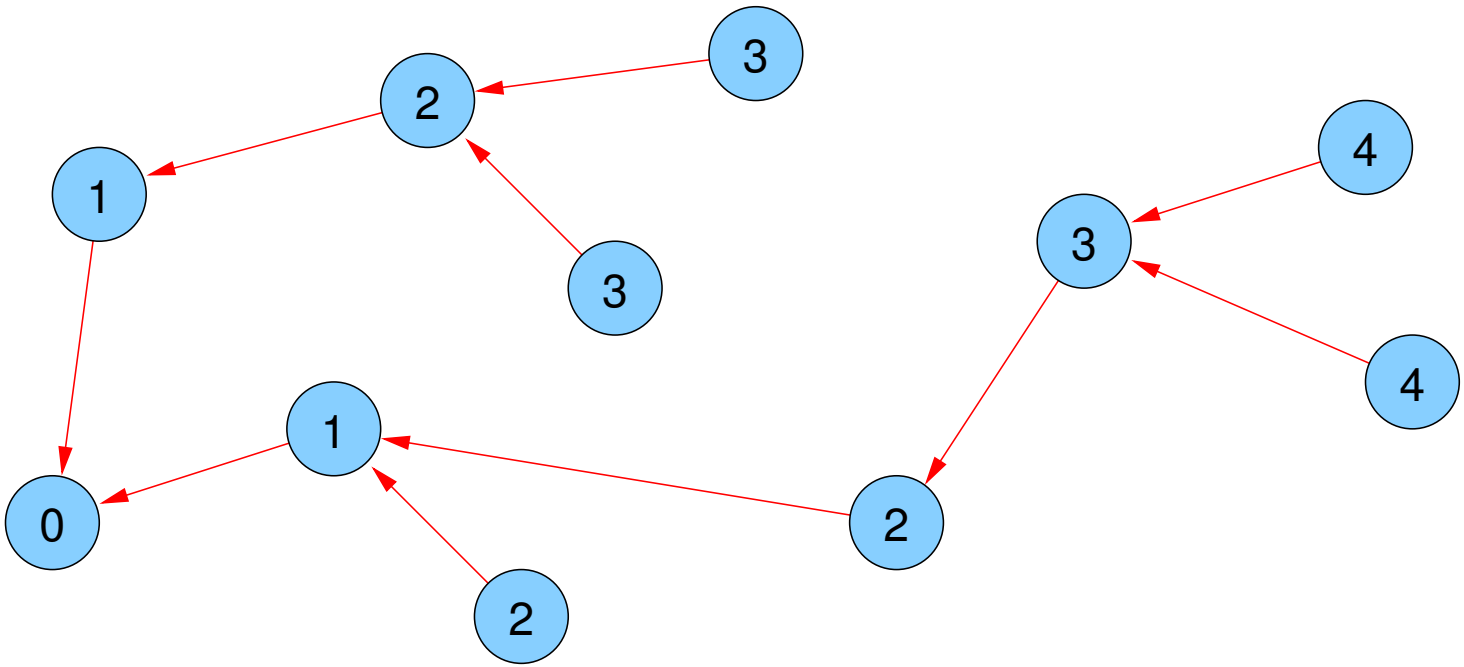
Breitensuche

- $d(v)$: Distanz von Knoten v zu s ($d(s) = 0$)



Breitensuche

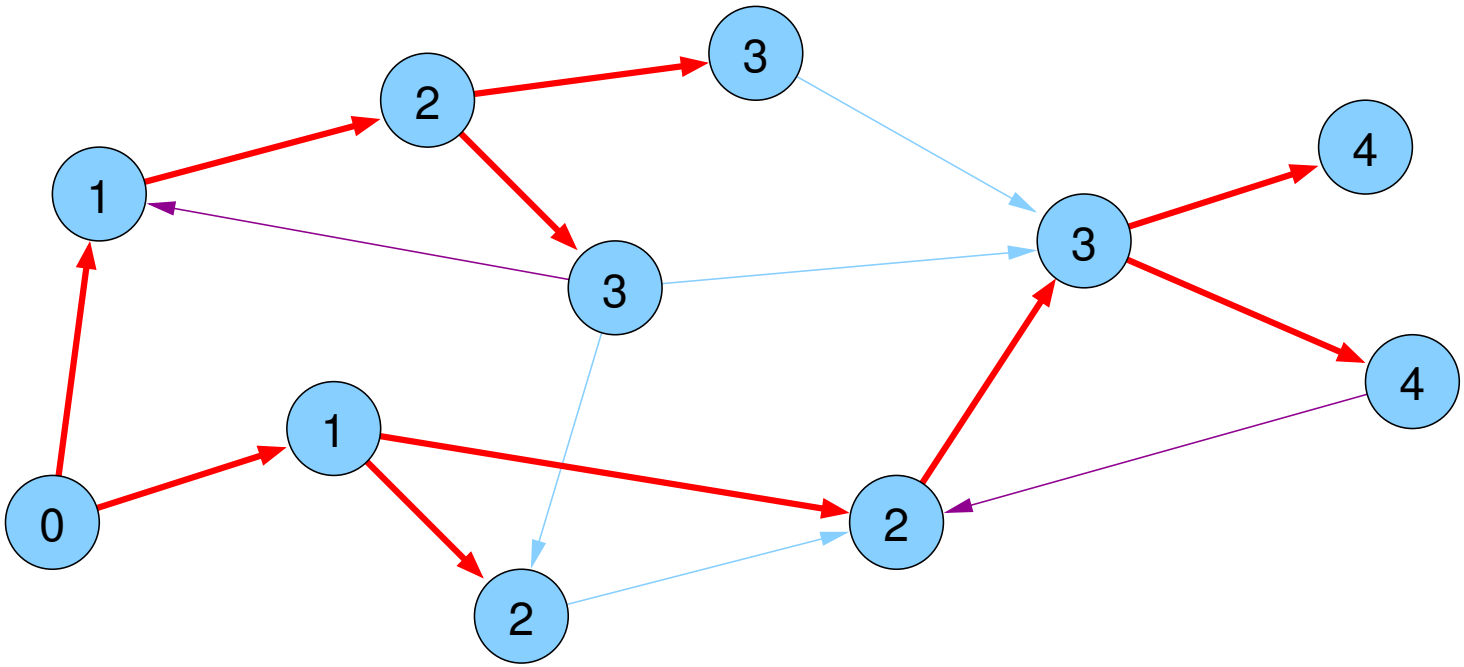
- parent(v): Knoten, von dem v entdeckt wurde
- parent wird beim ersten Besuch von §v gesetzt ($\Rightarrow$ eindeutig)



Breitensuche

Kantentypen:

- Baumkanten: zum Kind
- Rückwärtskanten: zu einem Vorfahren
- Kreuzkanten: sonstige



Breitensuche

```
void BFS(Node s) {
    d[s] = 0;
    parent[s] = s;
    List<Node> q = ⟨s⟩;
    while (!q.empty()) {
        u =q.popFront();
        foreach ((u, v) ∈ E) {
            if (parent[v] == null) {
                q.pushBack(v);
                d[v] = d[u]+1;
                parent[v] = u;
            }
        }
    }
}
```