
Effiziente Algorithmen und Datenstrukturen I

Aufgabe 1

Let $G = (V, E)$ be an undirected, simple, and connected graph with $|V| \leq |E|$ and a weight function $w : E \rightarrow \mathbb{R}$.

- Let T be a minimum spanning tree of G . Show that there exist two edges $e \in T$ and $f \notin T$ s.t. $(T \setminus e) \cup f$ is the second best minimum spanning tree of G .
- Describe an efficient algorithm that calculates the second best minimum spanning tree of G .

Hint: If one considers all spanning trees of some graph G and sorts them in ascending order based on their cumulative weight, then the first spanning tree in this sorted sequence is the minimum spanning tree and the second spanning tree in this sorted sequence is the second best minimum spanning tree.

Aufgabe 2

Consider the Improved JP-Algorithm and argue that the following upper bounds for the runtime of the operations are correct.

- deleteMin: $O(n_i)$
- delete: $O(m)$
- insert: $O(m)$
- decreaseKey: $O(m)$

Aufgabe 3

Remember from the lecture that it was shown that the improved runtime for Matrix-Vector Multiplication is $O(n \cdot m / \log n)$. Show that the method can be somewhat further improved to achieve a runtime of $O(n \cdot m / \log \max(m, n))$ (remember we are considering only binary matrices).

Aufgabe 4

Work through the *bcomb(int)* function that was discussed in the lecture in conjunction with the *4-Russen-Algorithmus* on the following binary matrix:

1	0	0	1	0
0	0	1	1	0
1	0	1	1	0
0	0	0	1	1
1	1	1	1	0