
Fundamental Algorithms

Deadline: December 5, 2007

Problem 1 (10 Points)

A binary tree is *full* if all of its vertices have either zero or two children. Let B_n denote the number of full binary trees with n vertices.

1. By drawing out all full binary trees with 3, 5, or 7 vertices, determine the exact values of B_3 , B_5 , and B_7 . Why have we left out even numbers of vertices, like B_4 ?
2. For general n , derive a recurrence relation for B_n .

Problem 2 (10 Points)

Review all the sort algorithms taken in the class. Compare their strength and weakness. If possible, try to explain them with day-to-day examples.

Prove that the lower bound for sorting is $n \lg n$

Problem 3

Stacks and Queues.

1. Write pseudo code for `push(x)`, `pop()`, `add(x)`, `delete()`.
2. How can one simulate a queue with two stacks! (no counting)

What is a circular queue?

Problem 4

Design the functions `insert(x)`, `search(x)` and `delete(x)` in a binary search tree – RECURSIVELY.

Compare the complexity with the iterative implementations.