1D quantum rings and persistent currents

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Motivation

In the last decades there was a growing interest for such microscopic systems like quantum rings because of few aspects:

- New physics behind quantum rigs, like Aharonov-Bohm-Effect and the possibility of controlling many-body systems with a well defined number of charge-carriers
- Possibility of the experimental realization of such systems, especially semiconductor technology
- developing new electronic devices

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Physical idea

- Place few to hundreds of electrons on a quasi 1D ring structure
- Study the effects of magnetic field through the ring: QM-currents periodic in magnetic flux Φ with the period $\Phi_0 = \frac{hc}{e}$, Aharonov-Bohm-Effect
- Study the effects of an impurity positioned on the ring: 1D-Wigner crystal, pinning of Wigner crystal (Marc's part)

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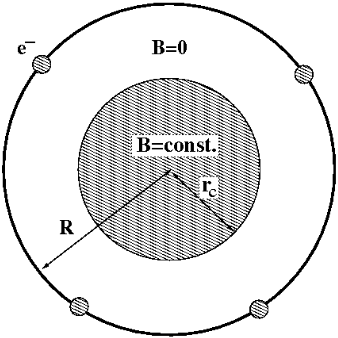
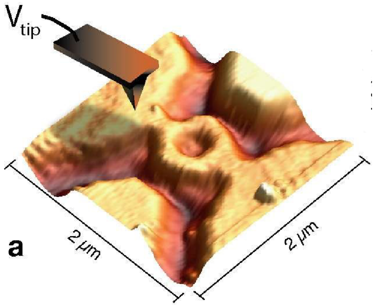
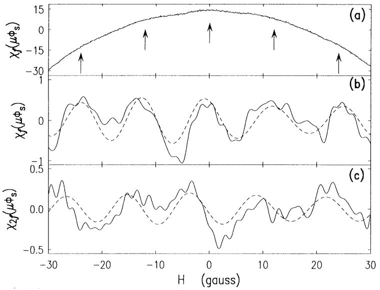
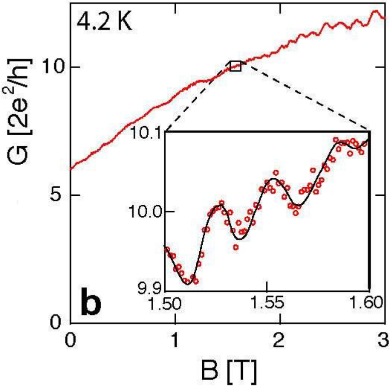

e⁻
B=0
B=const.
r_c
R


V_{tip}
a
2 μm
2 μm


(a)
(b)
(c)
H (gauss)


4.2 K
G [2e²/h]
B [T]
b

Outline

Theoretical modelling

Assumptions:

- Strictly one-dimensional ring filled with electrons
- Non-interacting particles
- Spinless fermions (electrons): only 1 particle per state
- Switching magnetic field on, such that finite flux through the ring, but field-free electrons

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Solving the Schrödiger-equation

1-particle Hamiltonian in cylindrical coordinates depends only on the polar angle φ:

$$H = -\frac{\hbar^2}{2m_e R^2}\frac{\partial^2}{\partial\varphi^2}$$

Which yields and the solution of the corresponding Schrödinger-equation:

$$\Psi_m(\varphi) = e^{im\varphi}, E_m = \frac{\hbar^2 m^2}{2m_e R^2}$$

where m is the angular momentum of the electron. $m \in \mathbb{Z}$ due to boundary conditions $\Psi(\varphi + 2\pi) = \Psi(\varphi)$. E_m the corresponding eigenenergy.

Many-particle states and energies

Since we assumed non-interacting particles, the many-body wavefunction of N electrons obeying the Pauli exclusion principle is given by the Slater-determinant:

$$\Psi_{m_1,\dots,m_N}(\varphi_1,\dots,\varphi_N) = \begin{vmatrix} \Psi_{m_1}(\varphi_1) & \Psi_{m_2}(\varphi_1) & \dots & \Psi_{m_N}(\varphi_1) \\ \Psi_{m_1}(\varphi_2) & \Psi_{m_2}(\varphi_2) & \dots & \Psi_{m_N}(\varphi_N) \\ \vdots & & & \vdots \\ \Psi_{m_1}(\varphi_N) & \Psi_{m_2}(\varphi_N) & \dots & \Psi_{m_N}(\varphi_N) \end{vmatrix}$$

$$\Psi_{m_k}(\varphi) = exp(im_k\varphi)$$

where $\Psi_{m_k}(\varphi)$ is given by the single-particle wave-function with angular momentum m_k.

Many-particle states and energies

This yields the (trivial) total energy...

$$E = \sum_{i}^{N} E_{m_i} = \sum_{i}^{N} \frac{\hbar^2 m_i^2}{2 m_e R^2}$$

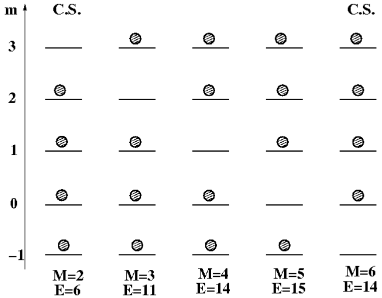
... and the total angular momentum:

$$M = \sum_{i}^{N} m_i$$

Thus, the lowest energy at a given total angular momentum M is obtained by occupying the single particle states next to each other.

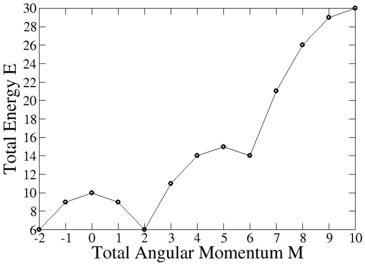
Many-particle states and energies

Occupation scheme of four electrons



An example of occupation of single particle states by four electrons

Energy spectrum of four electrons



The lowest possible energy E of four electrons at a given total angular momentum M

Outline

Schrödinger-equation including magnetic field

Choose a vector potential $\vec{A} = A_\varphi \cdot \vec{e_\varphi} + A_r \cdot \vec{e_r} + A_z \cdot \vec{e_z}$, such that

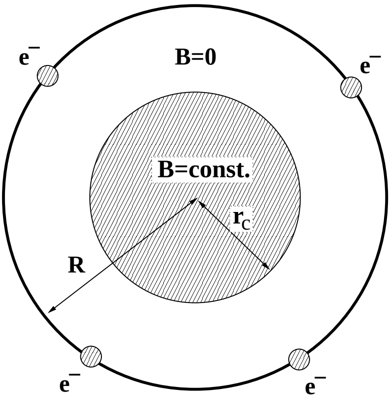
$$A_r = A_z = 0, \qquad A_\varphi = \begin{cases} \frac{B \cdot r}{2}, \text{if } r \leq r_c \\ \frac{B \cdot r_c^2}{2r} = \frac{\Phi}{2\pi r}, \text{if } r > r_c \end{cases}$$

since $\vec{B} = \vec{\nabla} \times \vec{A}$, this results in a magnetic field in z direction

$$B_z = \begin{cases} B, \text{if } r \leq r_c \\ 0, \text{if } r > r_c \end{cases}$$

while $B_\varphi = B_r = 0$.

Schrödinger-equation including magnetic field



The electrons are at $r = R$, meaning they are field-free if $r_c < R$. Nevertheless there is a magnetic flux $\Phi = \pi r_c^2 B$ penetrating the ring

Schrödinger-equation including magnetic field

The Hamiltonian with magnetic field can be written as

$$\hat{H} = \frac{1}{2m_e}(\hat{p} - e\hat{A})^2 = \frac{1}{2m_e}\left(-\frac{i\hbar}{R}\frac{\partial}{\partial\varphi} - \frac{e\Phi}{2\pi Rc}\right)^2$$

The eigenfunctions are still the same as for the field-free Hamiltonian, but the eigenenergies changed:

$$\Psi_m(\varphi) = e^{im\varphi}$$

$$E(m, \Phi) = \frac{\hbar^2}{2m_eR^2}\left(m - \frac{\Phi}{\Phi_0}\right)^2$$

where $\Phi_0 = \frac{hc}{e}$ is the flux quantum.

Energy spectrum

The ground state energy of the spectrum

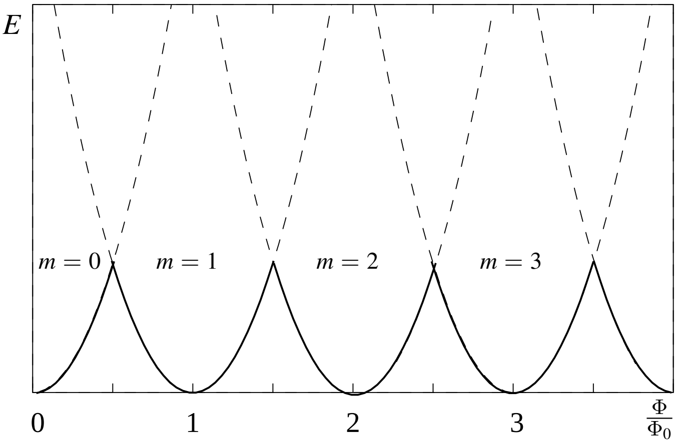
$$E(m,\Phi) = \frac{\hbar^2}{2m_e R^2}\left(m - \frac{\Phi}{\Phi_0}\right)^2$$

ia periodic in flux with the period Φ_0 in the following sense:

$$\frac{\Phi}{\Phi_0} \to \frac{\Phi}{\Phi_0} + 1$$

$$m \to m + 1$$

Energy spectrum



Persistent current

Now lets have a look at the current inside the ring. The current-density operator is given by:

$$\hat{\vec{j}} = \frac{c}{e}\frac{\partial \hat{H}}{\partial \vec{A}}$$

$$\vec{A} = \frac{\Phi}{2\pi R}$$

Thus, the derivative of energy with respect to flux is

$$\begin{aligned} -c\frac{\partial E}{\partial \Phi} &= c \cdot \langle\Psi| \frac{-\partial \hat{H}}{\partial \Phi} |\Psi\rangle = c \cdot \langle\Psi| \frac{-\partial \hat{H}}{\partial \vec{A}} \frac{\partial \vec{A}}{\partial \Phi} |\Psi\rangle = \\ &= \langle\Psi| \hat{\vec{j}}_\varphi \frac{-e}{2\pi R} |\Psi\rangle = \frac{-e}{2\pi R} \int_0^{2\pi} d\varphi j(\varphi) = I \end{aligned}$$

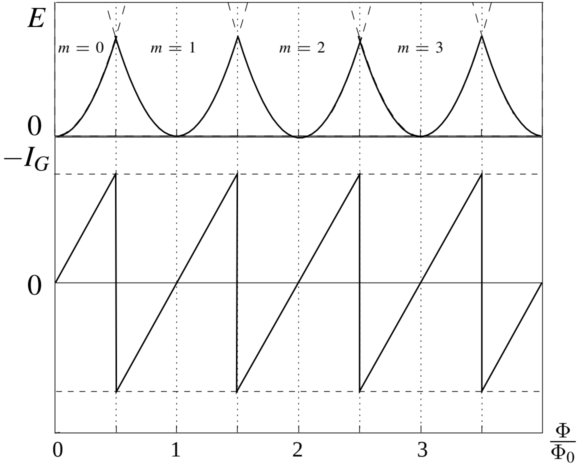
Persistent current

Thus, if the ground state energy E_G is periodic in the flux Φ, then the current

$$I_G = -c\frac{\partial E_G}{\partial \Phi}$$

is also periodic in Φ with the period Φ_0 !!!

Persistent current



E_G (upper diagram) and $I_G = -c\frac{\partial E_G}{\partial \Phi}$ (lower diagram) are both periodic in Φ with the period Φ_0

Alternative solution

Alternatively, lets chose a unitary transformation $\hat{U}$

$$\hat{U}(\varphi) = exp(\frac{-ie}{\hbar c}\int_0^{\varphi}\vec{A}\cdot d\vec{l}) = exp(\frac{-ie}{\hbar c}\frac{\Phi\varphi}{2\pi})$$

which transforms the Hamiltonian to field-free Hamiltonian in the following way:

$$\hat{H} \rightarrow \hat{H}' = \hat{U}\hat{H}\hat{U}^{-1} = -\frac{\hbar^2}{2m_e R^2}\frac{\partial^2}{\partial\varphi^2}$$

Lets apply $\hat{U}$ on the linear Schrödinger equation:

$$\begin{aligned} \Psi \rightarrow \Psi' &= \hat{U}\Psi \\ \hat{H} \rightarrow \hat{H}' &= \hat{U}\hat{H}\hat{U}^{-1} \end{aligned}$$

Alternative solution

This yields that for a given eigenfunction Ψ with $\hat{H}\Psi = E\Psi$

$$\hat{H}'\Psi' = \hat{U}\hat{H}\hat{U}^{-1}\hat{U}\Psi = E\Psi'$$

the eigenvalues are conserved. Since the transformed Hamiltonian is

$$\hat{H}' = -\frac{\hbar^2}{2m_e R^2}\frac{\partial^2}{\partial\varphi^2}$$

the solutions $\Psi'(\varphi)$ for the corresponding Schrödinger-equation are the already known eigenfunctions $\Psi'(\varphi) = exp(ik\varphi)$.

Alternative solution

Though the new eigenfunctions $\Psi'(\varphi) = exp(ik)\varphi$ are very similar the boundary conditions has changed. If periodic boundary conditions were true for the original wavefunction

$$\Psi(\varphi + 2\pi) = \Psi(\varphi)$$

then "twisted boundary conditions" are true for the new wavefunction:

$$\begin{aligned}\Psi'(\varphi + 2\pi) &= \hat{U}(\varphi + 2\pi)\Psi(\varphi + 2\pi) = \\ &= exp(\frac{-ie}{\hbar c}\frac{\Phi(\varphi + 2\pi)}{2\pi})\Psi(\varphi + 2\pi) = \\ &= \Psi'(\varphi) \cdot exp(\frac{-ie}{\hbar c}\Phi)\end{aligned}$$

Alternative solution

Thus, if $\Psi'(\varphi) = exp(ik\varphi)$ (k not necessarily an integer):

$$\Psi'(\varphi + 2\pi) = \Psi'(\varphi)exp(ik2\pi) \stackrel{!}{=} \Psi'(\varphi) \cdot exp(\frac{-ie}{\hbar c}\Phi)$$

$$\Rightarrow 2\pi k = -\frac{e}{\hbar c \Phi} + 2\pi m$$

where m can now be any integer. Then $\Psi'(\varphi)$ and $\Psi(\varphi)$ are

$$\Psi'(\varphi) = \exp(i\varphi(m - \frac{e\Phi}{\hbar c 2\pi}))$$

$$\Psi(\varphi) = \hat{U}^{-1}\Psi(\varphi) = \exp(i\varphi(m - \frac{e\Phi}{\hbar c 2\pi}) + \varphi\frac{ie\Phi}{\hbar c 2\pi}) = exp(im\phi)$$

Which is the already known solution resulting in a periodic ground state energy.

Alternative solution

The twisted boundary conditions result in the fact, that even if the electrons are in a field-free region, the wavefunction picks up a total phase Δ while moving along a closed path around the flux:

$$\Delta = -2\pi i \frac{\Phi}{\Phi_0}$$

This is also known as the Aharonov-Bohm-Effect.

Outline

Introduction

In order to study quantum rings properly one may need to experimentally control several parameters:

- Number of electrons on the ring
- The shape and size of the ring

The mechanisms of controlling these parameters of quantum rings are quite the same as for quantum dots, which are described by Leo Kouwenhoven and Charles Marcus in their paper "Quantum dots". The following sections are cited from there.

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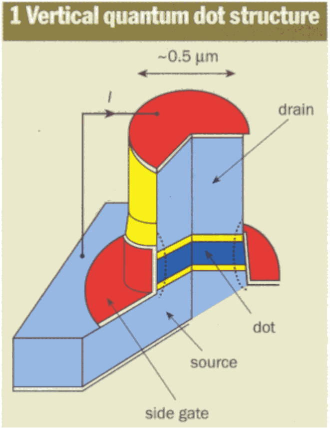
Coulomb blockade



The number of electrons in the dot and the shape of the confining potential are tuned via the side gate voltage. The current through the dot is tuned via the source-drain voltage.

Coulomb blockade

The mechanism which influence the number of electrons is the so-called Coulomb blockade. Given a certain number N of electrons on the dot, it takes some energy to overcome the Coulomb repulsion in order to bring one more electron on the dot. This amount of additional energy E_{add} can be described by a simple model, the constant-interaction model:

- The Coulomb interaction between the electrons is independent of N and given by the capacitance C of the dot
- Additional energy ΔE is needed, which is the difference of the quantum levels of the dot
- Therefore $E_{add} = \frac{e^2}{C} + \Delta E$ is the energy amount needed to place an electron on the dot

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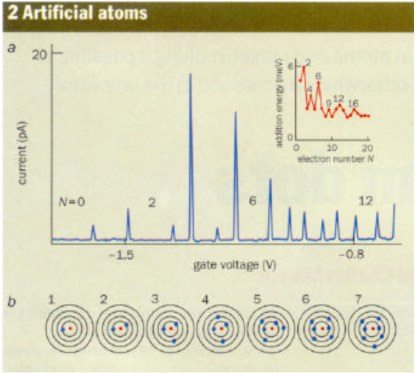
Coulomb blockade

This simple model explains quite accurate the following diagram for the current through the dot:



The larger energy amounts for the 3rd and the 7th electron are explained by quantum state energies of the dot.

Coulomb blockade

Let summarize:

- The quantum dot can be considered as an 2D artificial atom, since the electrons are squeezed in a flat region
- Its shell structure yields the quantum energy levels and therefore the energy needed to add more electrons
- The shell structure is very close to the one af a 2-dimensional circular symmetric harmonic potential
- The shell structure can be changed via applied voltages
- Therefore the number of electrons on the dot and its thermal fluctuation can be experimentally controlled

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Experiments

Experimental situation

- Plenty of experiments have been made on quantum rings and dots in order to study artificial atoms
- Presentation of an experiment on quantum rings made by T.Ihn, A. Fuhrer et. al. and described in their paper "Marvellous things in marvellous rings: energy spectrum, spins and persistent currents" from 2002
- Presentation of an experiment by V. Chandrasekhar et. al. on persistent currents in quantum rings made in 1991

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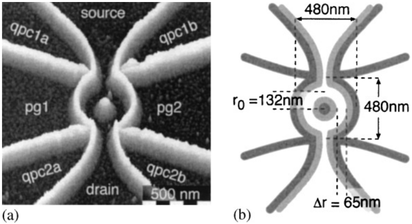
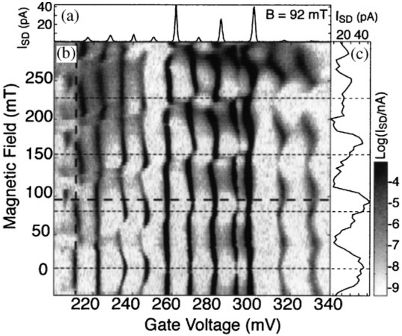
Experiments

T.Ihn et. al. oxidized a ring upon a AlGaAs/GaAs surface via an atomic force microscope (AFM) and depleted a 2D electron gas below the oxidized region. Here an AFM picture of the ring.



Quantum ring: a) AFM picture, bright oxidized regions separate conductive regions
b) Schematic sketch of the ring

Experiments



a) Greyscale plot of the source-drain current at constant $V_{SD} = 20\mu V$
b) Current at constant magnetic field $B = 92mT$
c) Current at constant side gate voltage $V_G = 200mV$

Outline

Persistent current in a gold loop

V.Chandrasekhar et. al. managed to measure the magnetic response of a single, isolated gold loop penetrated by a magnetic flux in 1991, which was another experimental evidence of persistent currents.

- Loops fabricated on Si substrates
- Cooling down to 300mK
- Measurement on three different gold loops: Two rings with diameters 2.4 and 4.0μm and a 1.4$\mu m \times 2.6\mu m$ rectangle
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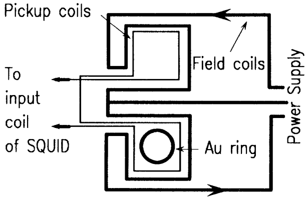

- DC current through the field coils generating a magnetic flux of $24 - 35G$ through the ring
- No current induced by magnetic flux generated by the DC current in the pickup coil due to the geometry
- Persistent current induced in the Au loop
- Almost no coupling of the Au-loop-magnetic field to the pickup coil due to inductance $\simeq pH$

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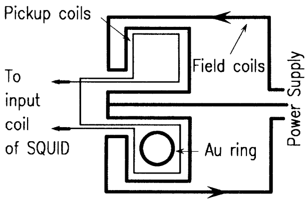

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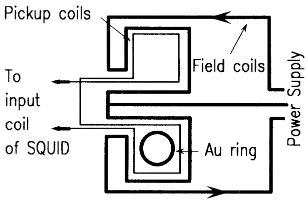

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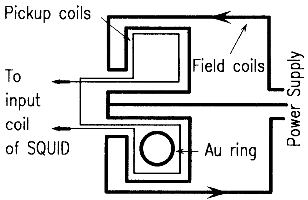

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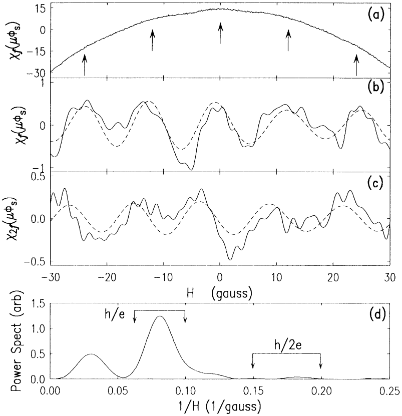
Here are the results of the measurement



Results

Lets summarize:

Theoretical model: assumptionsand results

- Non-interacting fermions on a 1D Ring
- Magnetic flux through the ring, field-free particles
- Persistent current periodic in flux Φ, interferometry effects

Experimental results

- Agreement with the theoretical prediction of periodicity of the current
- Interferometry effect at measurement of transport current through the ring

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- Interferometry effect at measurement of transport current through the ring

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- Magnetic flux through the ring, field-free particles
- Persistent current periodic in flux Φ, interferometry effects

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