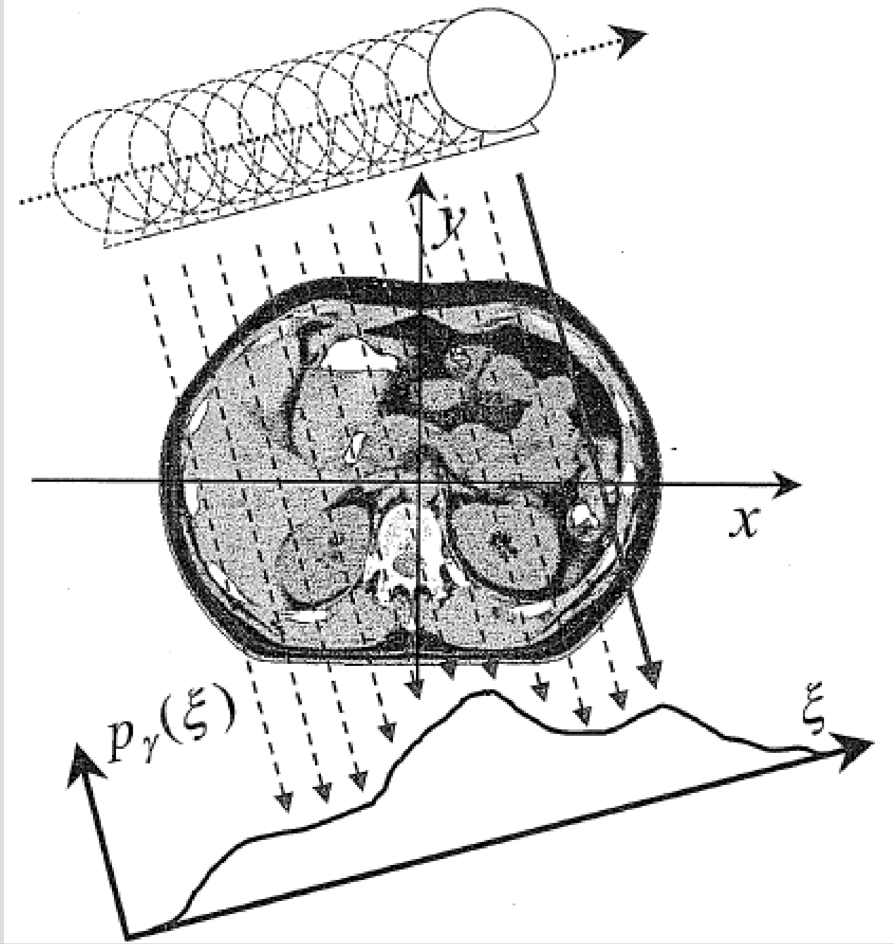
Line Integrals

- Line integrals represent the integral of some parameter of the object along the line (e.g. attenuation of x-rays)
  - Object: $f(x,y)$
  - Line: $x\cos\theta + y\sin\theta = t$
  - Line integral / Radon transform:

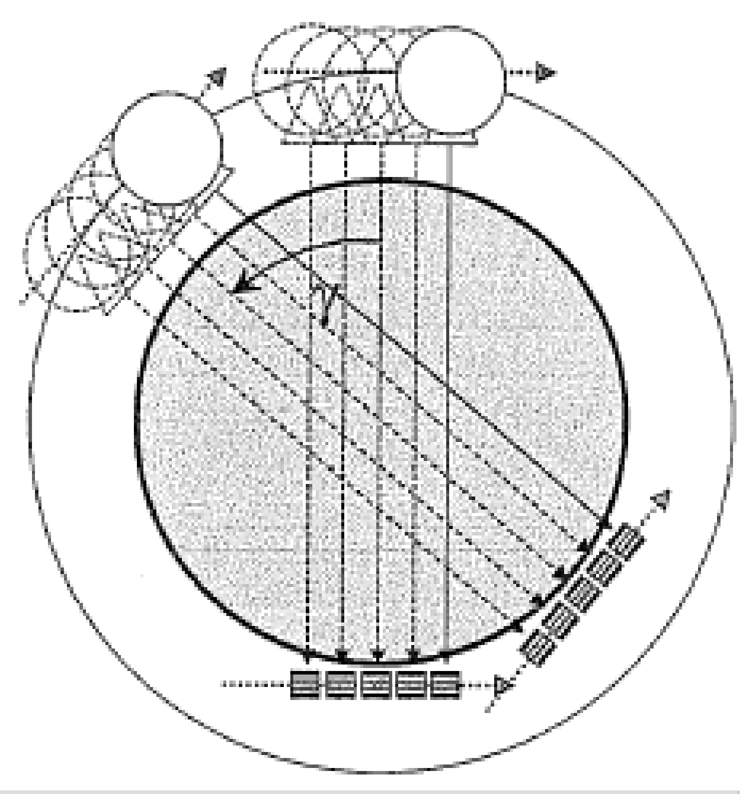
$$P_\theta(t) = \int_{(\theta,t)line} f(x,y)\,ds$$



- A set of line integrals form a projection

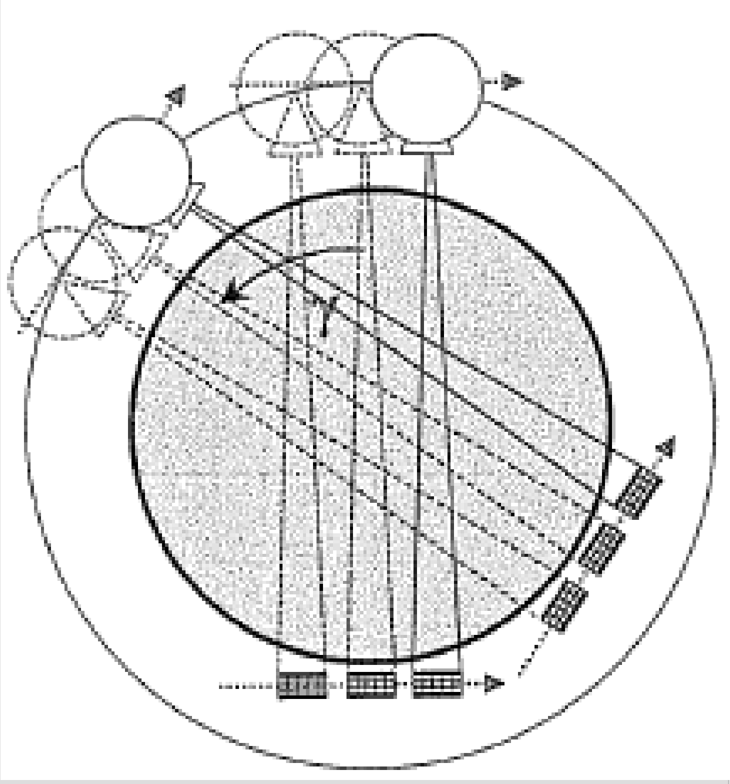
1st generation tomographs
Rotation-translation pencil beam

- Pencil beam, one detector
- For one angle a set of parallel line integrals is taken and form a projection. Then the angle is changed.



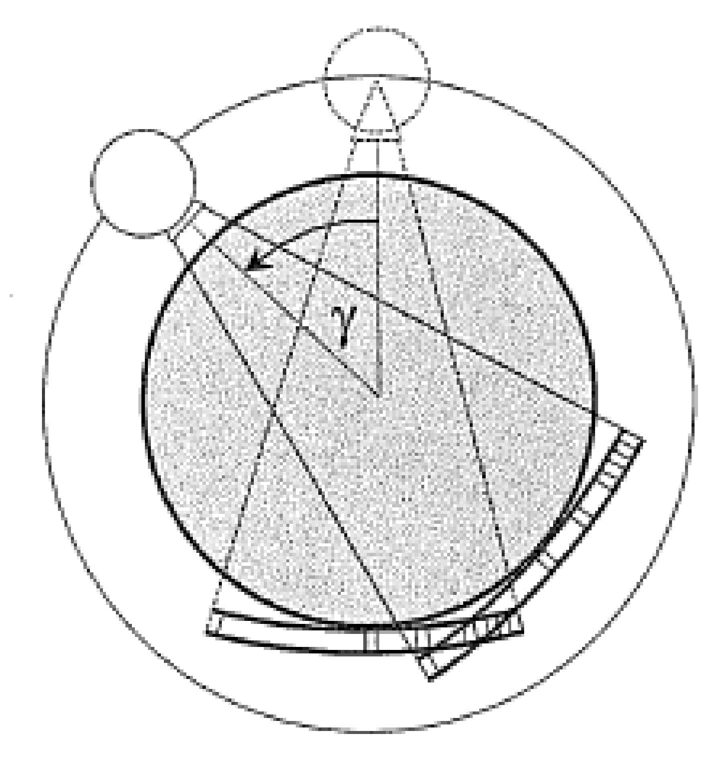
2nd generation tomographs Rotation-translation fan beam

- Fan beam (10°), ~30 detectors
- Multiple line integrals along a fan are taken simultaneously. Several parallel steps are done for different angles



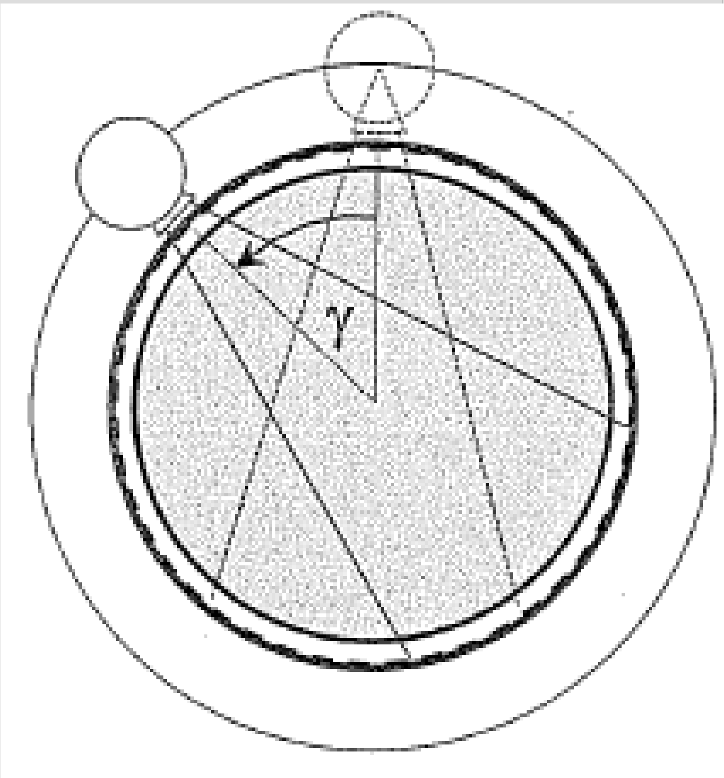
3rd generation tomographs Rotation-rotation single slices

- Fan beam (40°-60°), up to 1000 detectors
- A projection used only line integrals following a fan. All line integrals for one projection are taken simultaneously
- No translation is necessary



4th generation tomographs Rotation-fix closed detector array

- Fan beam (40°-60°), up to 5000 fixed detectors
- Rotation only, a translation is not necessary

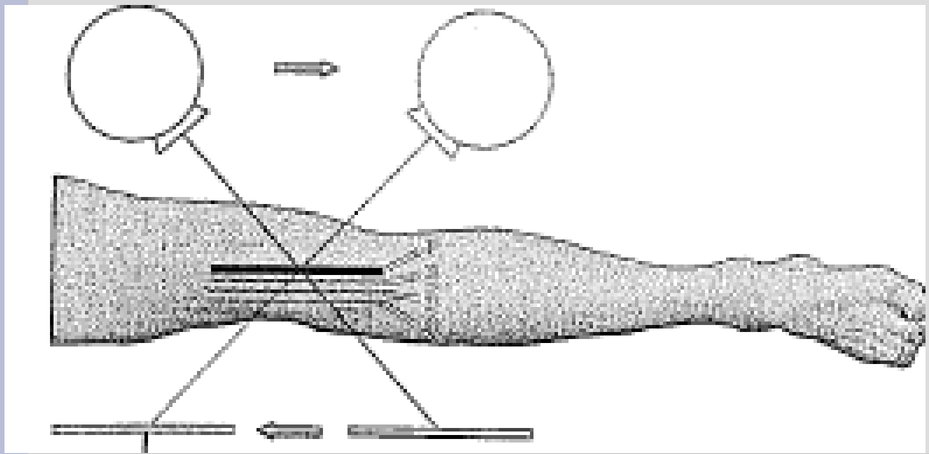


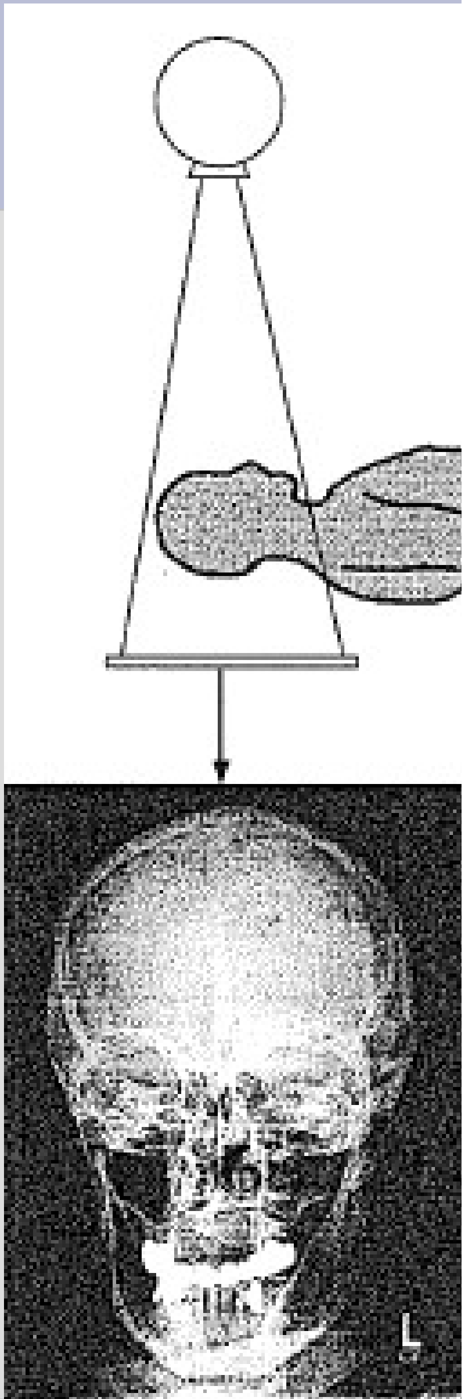
Purpose of tomography

- Conventional X-Ray provides only projections
  - no spatial information
  - averaging of all slices

    => low contrast

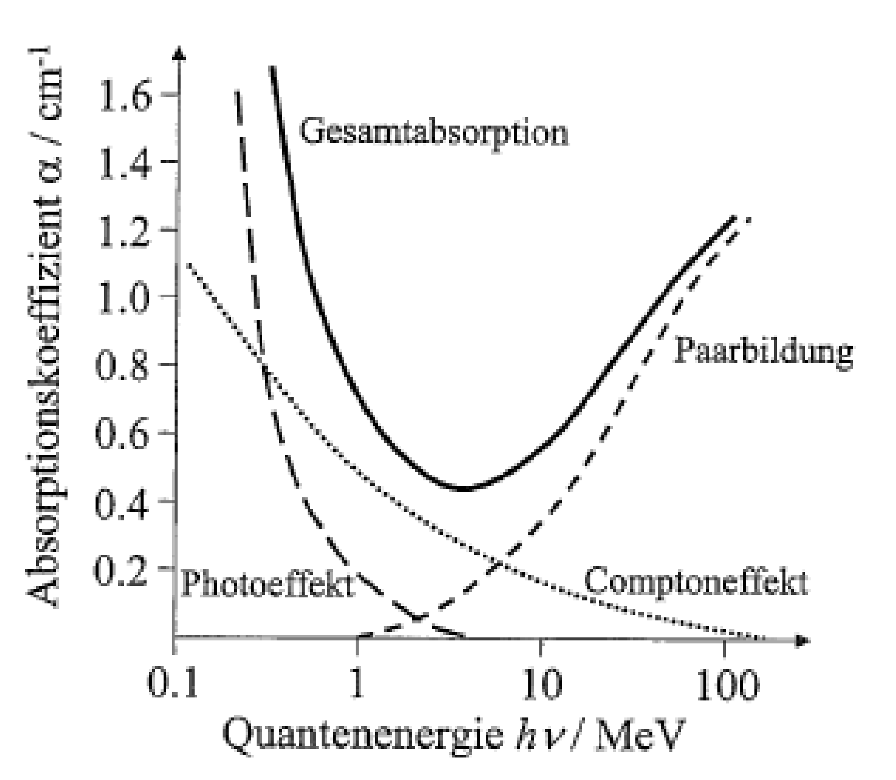
- Tomography tries to revert the projection

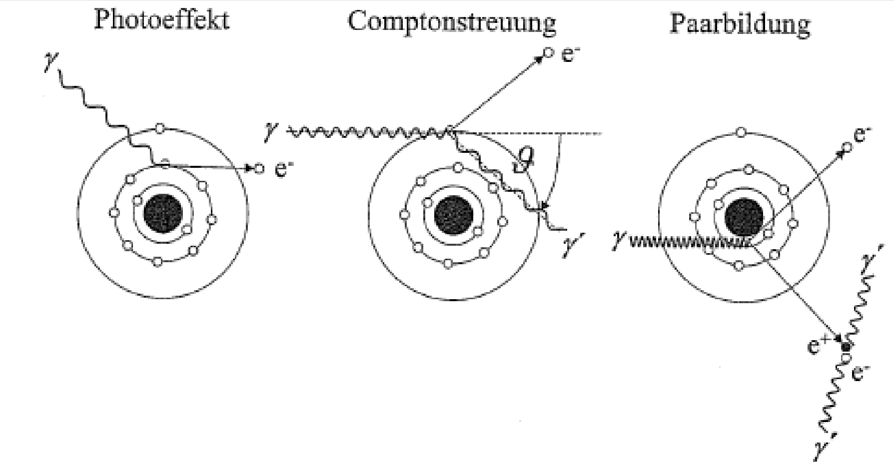




What is measured?

- The beam intensity and therewith the loss of beam intensity is measured
- Loss of intensity due to:
  - Photoelectric absorption
  - Compton effects
  - Pair production (PET)
- Loss is energy-dependent

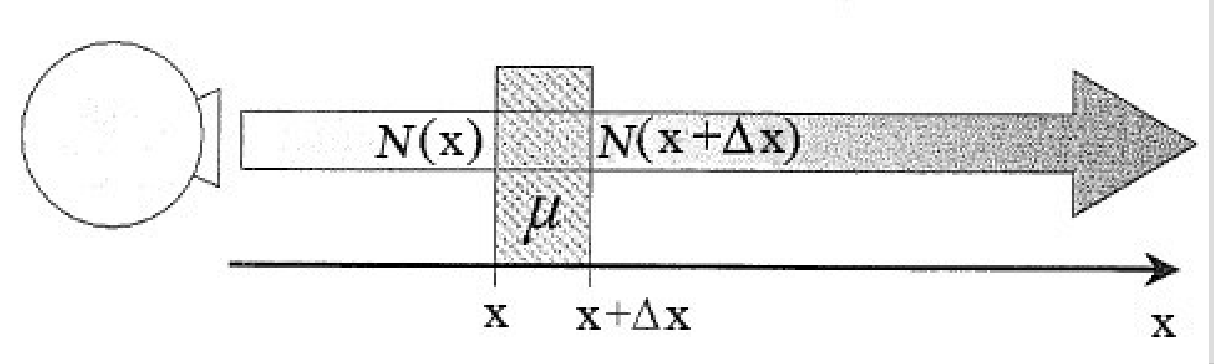




What is measured?

- μ is the attenuation coefficient of a material representing the photon loss rate due to the Compton effect and photoelectric absorption. μ is given by

$$\frac{\Delta N}{N} \cdot \frac{1}{\Delta x} = -\mu$$



What is measured?

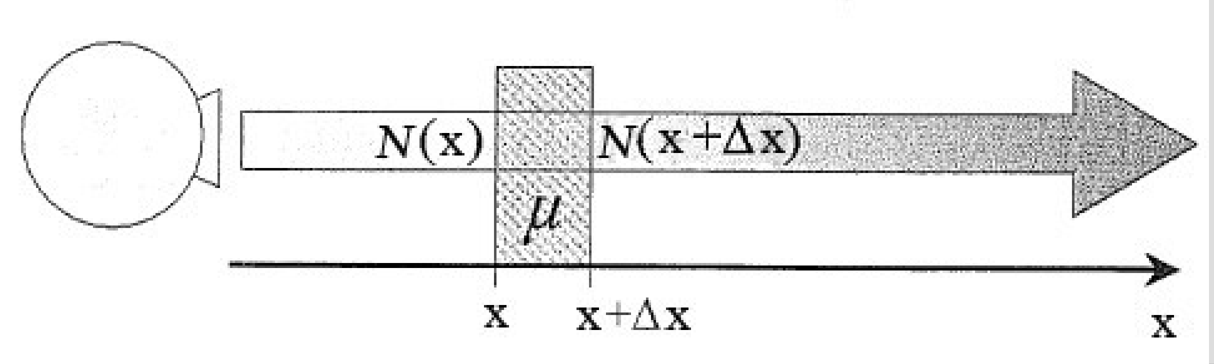
- The intensity after passing through Δx is given by:

$$N(x+\Delta x) = N(x) - \mu N(x)\Delta x$$

,whereas μ is assumed to be constant.

- As Δx goes to zero we obtain

$$\lim_{\Delta x \to 0} \frac{N(x+\Delta x) - N(x)}{\Delta x} = \frac{dN}{dx} = -\mu N(x)$$



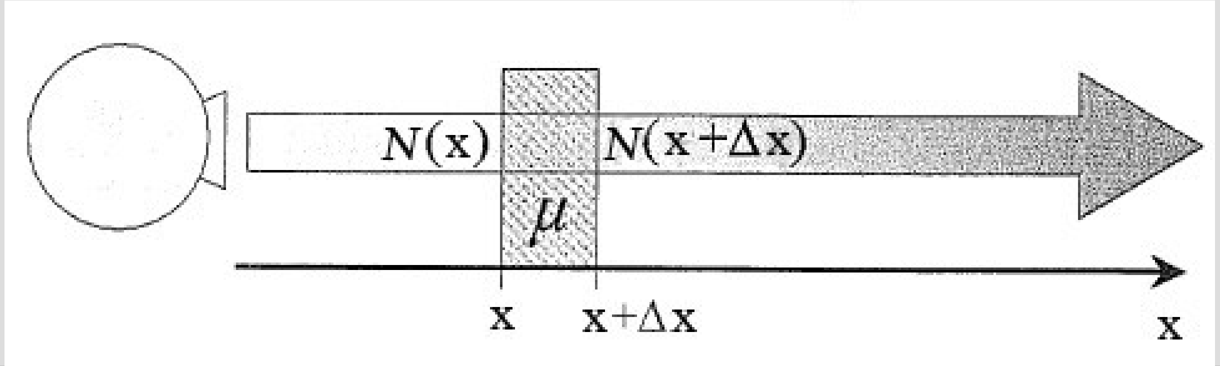
What is measured?

- Integration both sides

$$\int \frac{dN}{N(x)} = -\mu \int dx$$

,we obtain

$$\ln|N| = -\mu x + C$$



What is measured?

- The number of photons as function of the position is given by:

$$N(x) = N_{in} \exp\left[-\mu x\right]$$

,where N_{in} is the number of photons entering the object.

- This is the Lambert-Beer law
- This is only true for an x-ray beam consisting of monochromatic photons and a constant μ.

What is measured?

- $\mu(x,y)$ denotes the attenuation coefficient of a body
- The number of exiting photons is given by:

$$N_d = N_{in} \exp\left[-\int_{ray} \mu(x,y)ds\right]$$

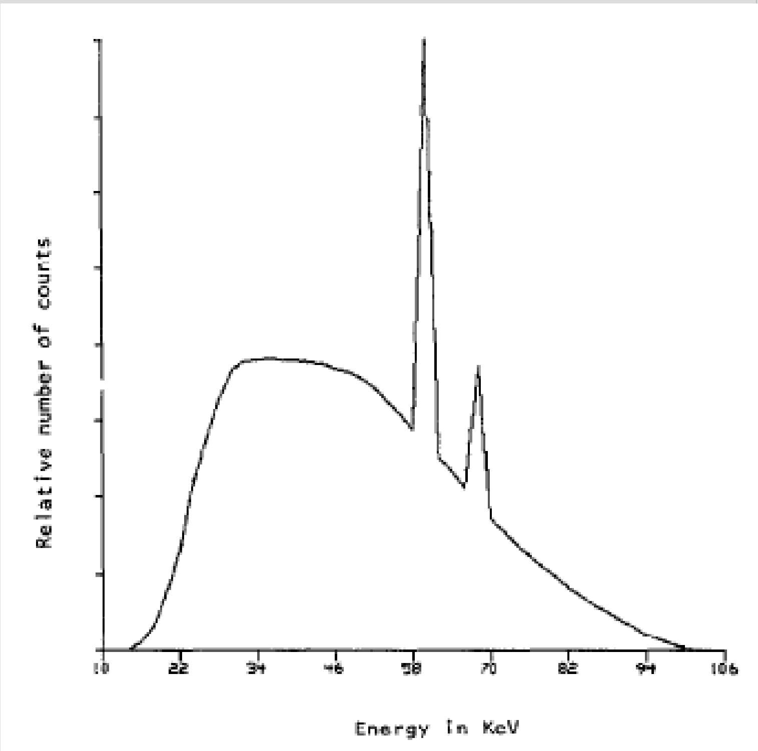
- This is only true for an x-ray beam consisting of monochromatic photons.

Monochromatic and polychromatic x-ray

- Monochromatic: X-ray beam consists only of photons with the same energy (in practice this type of x-ray is not used)
- Polychromatic: X-ray consists of a spectrum of photons with different energy

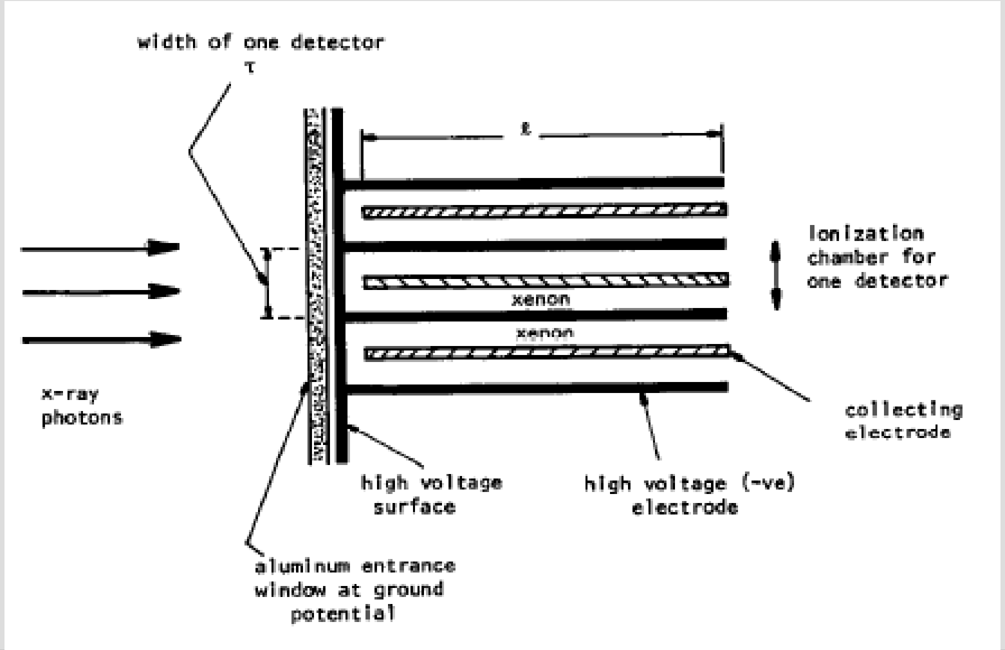
$$N_d = \int S_{in}(E) \exp\left[-\int \mu(x,y,E)\,ds\right] dE$$

- Beam hardening => Artifacts



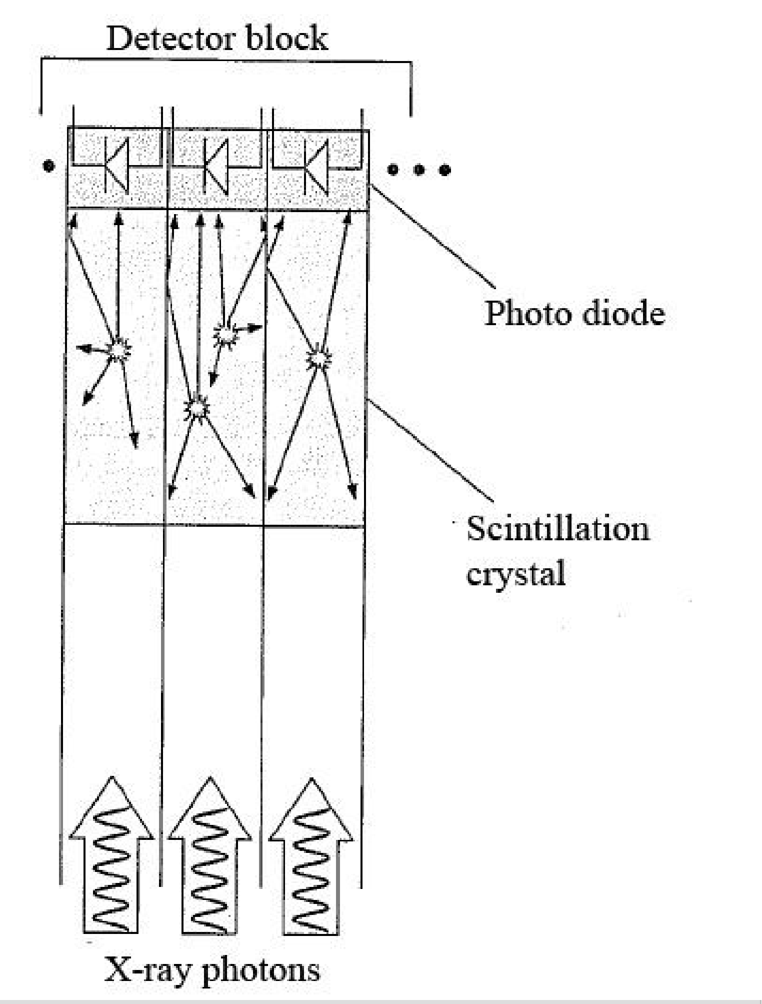
Detectors

- Xenon ionization detectors
- X-ray photons enter the detector chamber and ionize gas. The resulting current is measured.
- Used in 3rd generation tomographs



Detectors

- Scintillation detectors: Using a crystal the x-ray is transformed into photons with longer wavelength. These are measured using photo diodes.
- Used in 4th generation tomographs



Hounsfield scale

- Defined by Sir Godfrey Newbold Hounsfield
- 0 Hounsfield Units (HU) defined as radiodensity of distilled water
- -1000 HU defined as radiodensity of air
- Corresponds to the linear attenuation coefficient by:

$$H = \frac{\mu - \mu_{water}}{\mu_{water}} \times 1000$$

Substance	Approx. value
Bone	80-1000
Calcification	80-1000
Congealed blood	56-76
Grey matter	36-46
White matter	22-32
Water	0
Fat	-100
Air	-1000