JASS 2009 - Joint Advanced Student School, St.Petersburg, 29.03-7.4.2009
Numerical Simulation in Turbomachinery

Numerical Simulation of Pressure Surge with the Method of Characteristics

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Nomenclature

Roman Letters

A	$[m^2]$	area
a	$[ms^{-1}]$	wave velocity
D	$[m]$	diameter
E	$[Nm^{-2}]$	Young's modulus
F	$[N]$	force
g	$[ms^{-2}]$	gravitation constant
L	$[m]$	length
M	$[Nm]$	torque
p	$[Pa]$	pressure
P	$[W]$	power
R	$[m]$	radius
s	$[m]$	wall thickness
t	$[s]$	time
T	$[s]$	periodic time
v	$[ms^{-1}]$	velocity
x	$[m]$	coordinate

Greek Letters

Δ	$[-]$	difference
ϵ	$[-]$	strain
μ	$[-]$	Poisson ratio
ρ	$[kgm^{-3}]$	density
σ	$[Nm^{-2}]$	stress
τ	$[-]$	opening ratio
τ	$[Nm^{-2}]$	shear
ω	$[rads^{-1}]$	angular speed

Subscripts

a	axial
f	friction
F	fluid
g	gravity
max	maximum
p	pressure
P	pipe
ref	reference
R	reflection
Sys	system
T	turbine
W	surge chamber
φ	circumferential

1 Introduction

Pressure surge and pressure oscillations are a problem in every pipe system, even more when some active internals like pumps or turbines are involved. There are two categories of damage that can arise from surge events:

- catastrophic failure like bursting or buckling of the pipeline,
- fatigue failure of the pipeline, supports and of equipment components.

Both are undesired and should be avoided in every way. Therefore in an early stage of design of such systems all interactions of the components that lead to surges or pressure oscillations have to be known. Due to the complexity of the systems and the complexity of the phenomena arising from surge events, this information can only be provided by numerical simulation or very expensive experiments.
Therefore the need for efficient computer codes to simulate surge events is obvious. In this work an introduction to a model for the simulation of surge phenomena in hydraulic piping is given and applications of a computer code based on this model are presented.

2 Model

2.1 Governing Equations

In this section the governing equations characterizing surge phenomena are derived. For the investigation of surge phenomena in piping like in fig.1 the pressure p and the velocity v are the variables of interest.

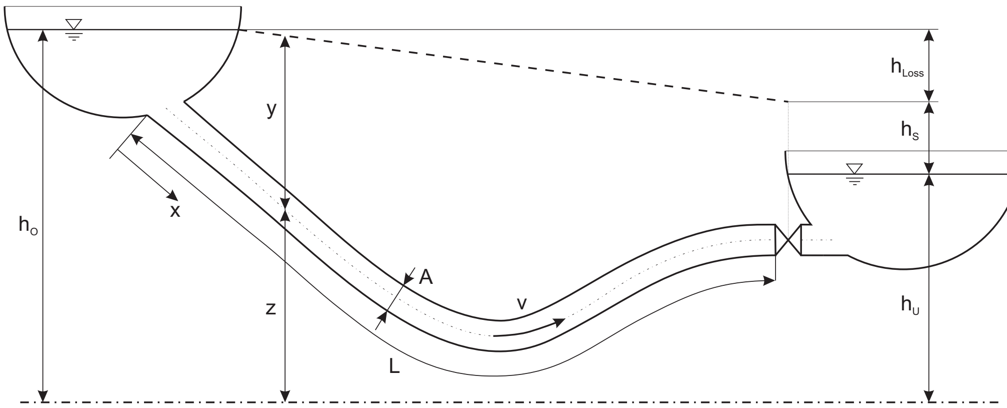

Figure 1: *Scheme of a simple piping system*

Due to the nature of surges and the characteristics of pipings the problem is assumed to be one-dimensional and unsteady. The compressibility of the fluid and the elasticity of the piping have to be taken into account.

2.1.1 Continuity Equations

Applying the conservation law of mass on the pipe shown in fig.2 the continuity equation (eq.2.1) can be derived:

$$\frac{\partial(\rho A)}{\partial t} + \frac{\partial(\rho v A)}{\partial x} = 0. \tag{2.1}$$

After taking into account the product rule and some simple conversions eq.2.1 can be written as:

$$A\frac{\partial \rho}{\partial t} + \rho\frac{\partial A}{\partial t} + Av\frac{\partial \rho}{\partial x} + \rho v\frac{\partial A}{\partial x} + \rho A\frac{\partial v}{\partial x} = 0. \tag{2.2}$$

Dividing 2.2 by ρA and with the definition of the total differential:

$$\frac{d}{dt} = \frac{\partial}{\partial t} + \frac{dx}{dt}\frac{\partial}{\partial x}, \tag{2.3}$$

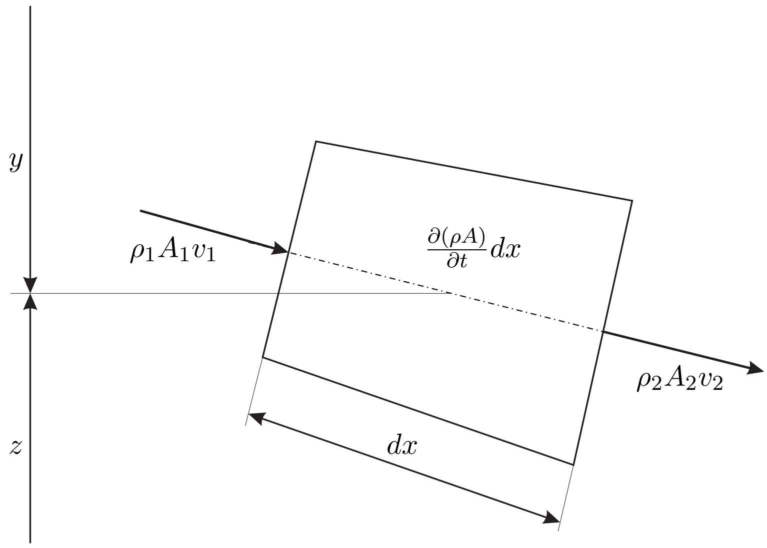

Figure 2: *Pipe section*

the continuity equation can by transformed to:

$$\frac{1}{\rho}\frac{d\rho}{dt} + \frac{1}{A}\frac{dA}{dt} + \frac{\partial v}{\partial x} = 0. \tag{2.4}$$

To obtain an equation in terms of the variables of interest (p and v), the derivative of ρ and A have to be eliminated from equation 2.4. Therefore a system elasticity E_{Sys} is defined. E_{Sys} contains the fluid elasticity and the elasticity of the pipe (E_F, E_P).
The fluid elasticity is the correlation of the pressure change and the relative change in volume:

$$E_F = \frac{dp}{\frac{dV}{V}}. \tag{2.5}$$

Due to the conservation of mass, the volume change in the control volume can be expressed as a change in density:

$$\frac{d\rho}{dt} = \frac{\rho}{E_F}\frac{dp}{dt}. \tag{2.6}$$

The derivative of A with respect to t can be linked to the change in pressure by using the mechanical model of a thin pipe under internal pressure.

Assuming a circular pipe with the radius R the change in area A can be written as:

$$\frac{dA}{dt} = 2\pi R\frac{dR}{dt}, \tag{2.7}$$

or in terms of the strain ϵ:

$$\frac{1}{A}\frac{dA}{dt} = 2\frac{d\epsilon}{dt}. \tag{2.8}$$

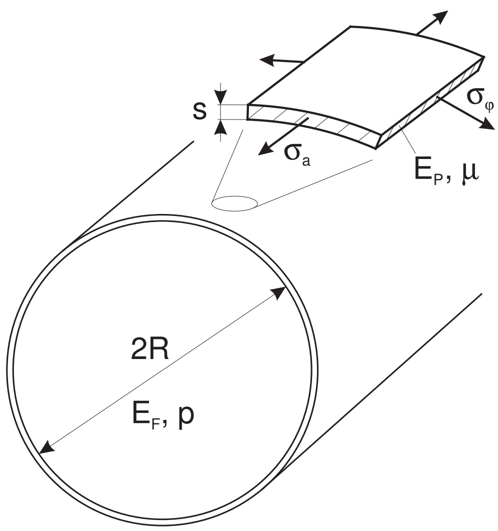

Figure 3: *Pipe section under internal pressure*

If Hook's law applies and the piping is fixed on both ends equation 2.9 holds:

$$\frac{1}{A}\frac{dA}{dt} = \frac{2}{E_P}(1-\mu^2)\frac{d\sigma_\varphi}{dt}. \tag{2.9}$$

If the wall thickness is small compared to the diameter D the stress σ_φ can be approximated with equation 2.10

$$\frac{d\sigma_\varphi}{dt} = \frac{R}{s}\frac{dp}{dt} \tag{2.10}$$

Inserting these expressions into equation 2.4 leads to

$$\frac{dp}{dt}\left[\frac{1}{E_F} + \frac{1}{E_P}\frac{2R}{s}(1-\mu^2)\right] + \frac{\partial v}{\partial x} = 0, \tag{2.11}$$

where the term in the brackets can be identified as the system elasticity E_{Sys}. Therewith the wave velocity in such a piping can be defined as:

$$a^2 = \frac{E_{Sys}}{\rho}. \tag{2.12}$$

Using expression 2.12 with 2.11 gives the continuity equation for a compressible fluid in an elastic piping:

$$\frac{\partial p}{\partial t} + v\frac{\partial p}{\partial x} + \rho a^2\frac{\partial v}{\partial x} = 0. \tag{2.13}$$

2.1.2 Momentum Equations

The momentum equation is derived from the conservation law of momentum:

$$\sum_i F_i = \rho A\left(v\frac{\partial v}{\partial x} + \frac{\partial v}{\partial t}\right). \tag{2.14}$$

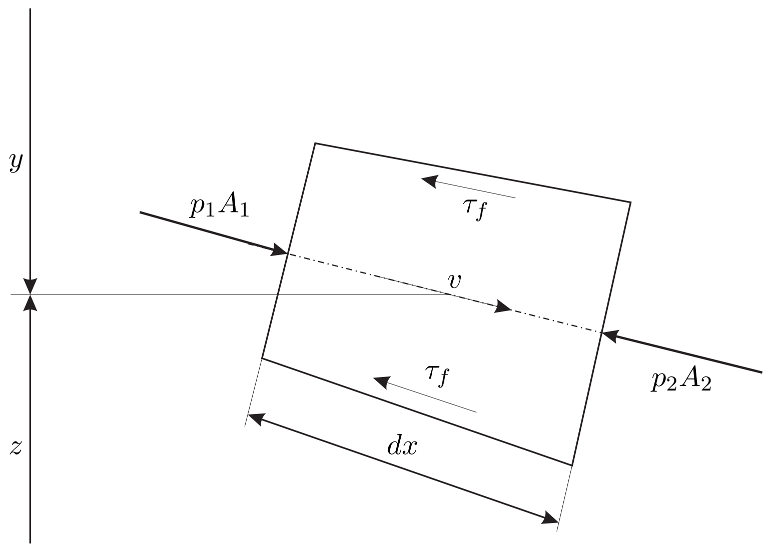

Figure 4: *Pipe section*

On the pipe section shown in fig.4 are acting a pressure force

$$F_p = -A\frac{\partial p}{\partial x}dx, \tag{2.15}$$

a gravity force

$$F_g = -\rho g A\frac{\partial z}{\partial x}dx, \tag{2.16}$$

and the friction force

$$F_f = -2\pi\ R\tau_f dx. \tag{2.17}$$

As the value of τ_0 is unknown, it will be approximated by the friction losses of a steady state flow with the same velocity:

$$\tau_f = \frac{\rho\lambda v|v|}{8}, \tag{2.18}$$

$$\lambda = \frac{4J_f gR}{v^2}. \tag{2.19}$$

Summed up and divided by $\rho A dx$ the final momentum equation is rewritten as:

$$\frac{1}{\rho}\frac{\partial p}{\partial x} + g\frac{\partial z}{\partial x} + gJ_f + v\frac{\partial v}{\partial x} + \frac{\partial v}{\partial t} = 0. \tag{2.20}$$

2.1.3 Final Set of Equations

Equation 2.11 and 2.20 characterize the unsteady, one dimensional and compressible flow in a piping. These two equations are a first order hyperbolic differential equation system (PDE). There are two dependent (p, v) and two independent (x, t) variables. A, a, ρ, g, J_e are parameters of the system.
The solution of such an equation system can be achieved by means of the Method of Characteristics.

2.2 Method of Characteristics

The Method of Characteristics is based on the conclusion that interferences propagate at a certain speed and on the existence of constant variables along characteristic lines. Along these characteristic lines the PDE becomes an ordinary differential equation which can be solved easily.
For the derivation and further information refer to [1], [2], [3] or [4].

2.3 Simplifications

For the implementation of the Model in the developed computer programm some simplifications are applied. Due to the fact that the wave velocity a is for most cases at least one magnitude above the convective speed v, it can be neglected.

$$v \ll a \tag{2.21}$$

The equation system is therewith simplified to:

$$\frac{\partial p}{\partial t} + \rho a^2 \frac{\partial v}{\partial x} = 0, \tag{2.22}$$

$$\frac{\partial v}{\partial t} + \frac{1}{\rho}\frac{\partial p}{\partial x} + gJ_f = 0. \tag{2.23}$$

In this case the characteristic lines are straight lines with constant slope for both directions.
As the velocity along the characteristic lines is not known the friction term has to be approximated:

$$\int_A^P J_f dt = J \int_A^P v^2 dt \simeq J v_A |v_A| \Delta t \tag{2.24}$$

For applications like hydropower plants with weak influence of friction a first order approximation (2.24) is sufficient [2].

3 Application

In this section some test cases and applications of the developed computer programm are presented.

3.1 Water hammer

If a valve at the downstream end of a pipe with steady flow is suddenly closed, the mass of water ahead of the closure is still moving forward with some velocity, thus building up high pressure and shock waves. This is known as a water hammer or Joukowski pressure surge.

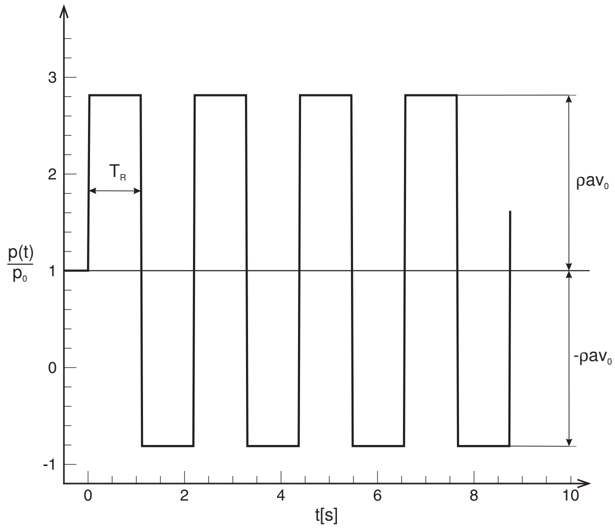

Figure 5: *Pressure at valve*

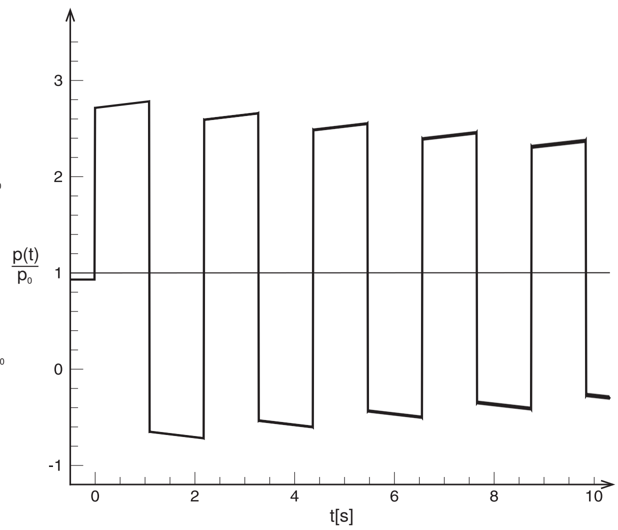

Figure 6: *Pressure at valve considering friction*

The maximum pressure that can be build through such a surge event can be calculated from the conservation laws:

$$\Delta p_{max} = \pm \rho a v_0 \, . \tag{3.1}$$

Fig.5 and 6 show the pressure over time at the valve. In fig.5 friction is neglected. Due to the consideration of friction in fig.6 a steady state solution is reached after some time.

3.2 Oscillating Valve

If the valve at the downstream end of the pipe is now opened and closed with certain frequencies, resonance occurs. In fig.7 the results of a simulation of a resonance case are shown. The opening $\tau = \frac{A}{A_{max}}$ is varied sinusoidally with a frequency of $\frac{2}{T_{Sys}}$. This leads to a standing wave in the pipe.

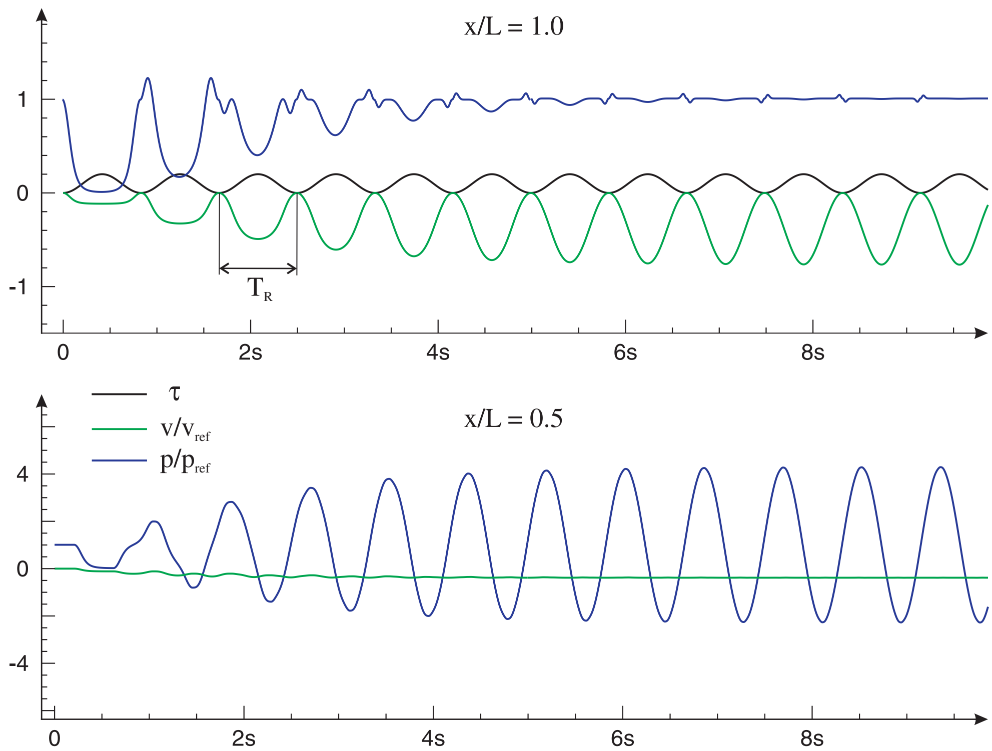

Figure 7: *Velocity and pressure in a pipe with oscillating valve excitation*

After a short period of transient oscillation a standing wave with a node at the valve and at the open end can be observed. Due to the damping caused by friction the amplitude of the oscillation is limited.

3.3 Surge chamber oscillation

Surge chambers are a common equipment of hydropower plants. Their main tasks are the hydraulic uncoupling of pressure tunnel and penstock, the reduction of pressure fluctuations and the improvement of control.
Surge chamber oscillations are induced by the interaction of high frequent surges and low frequent inertia waves. These oscillations can lead to severe problems in the power plant operations. Therefore surge chambers have to be well designed in order to reduce the effects of oscillations [5].

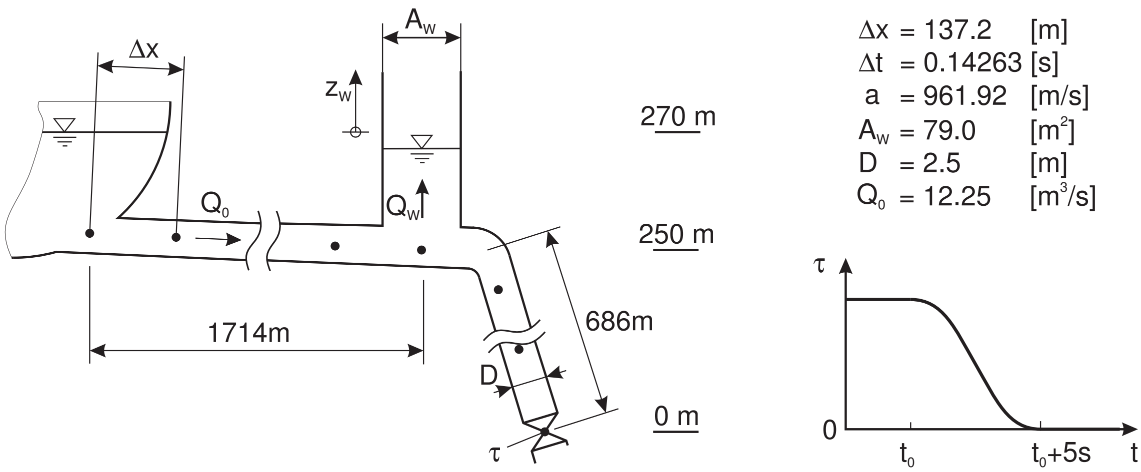

Figure 8: *Model*

In fig. 8 the computational model of a power plant with surge chamber is shown. Fig.9 and 10 present some results from the calculation performed on this model.

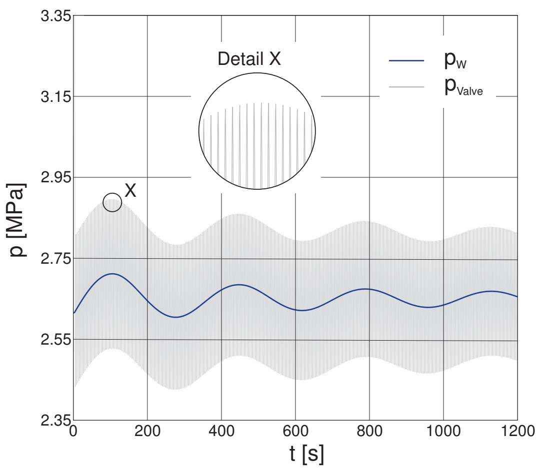

Figure 9: *Pressure in penstock*

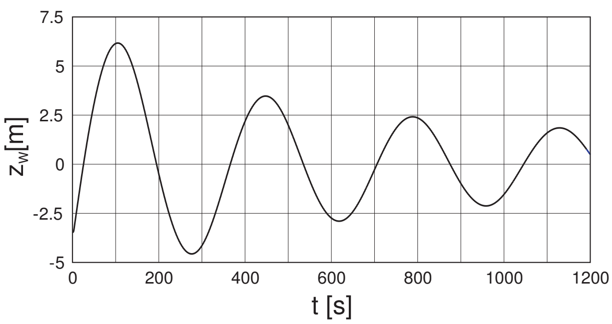

Figure 10: *Elevation of the water surface in the surge chamber*

3.4 Power plant

Turbine operation and turbine failure are one of the main causes of surges in the piping of hydropower plants. Phenomena arising from these surges are a typical example where a controlled event strongly depends on the interaction between the characteristics of the turbine, governor and the associated system [6].

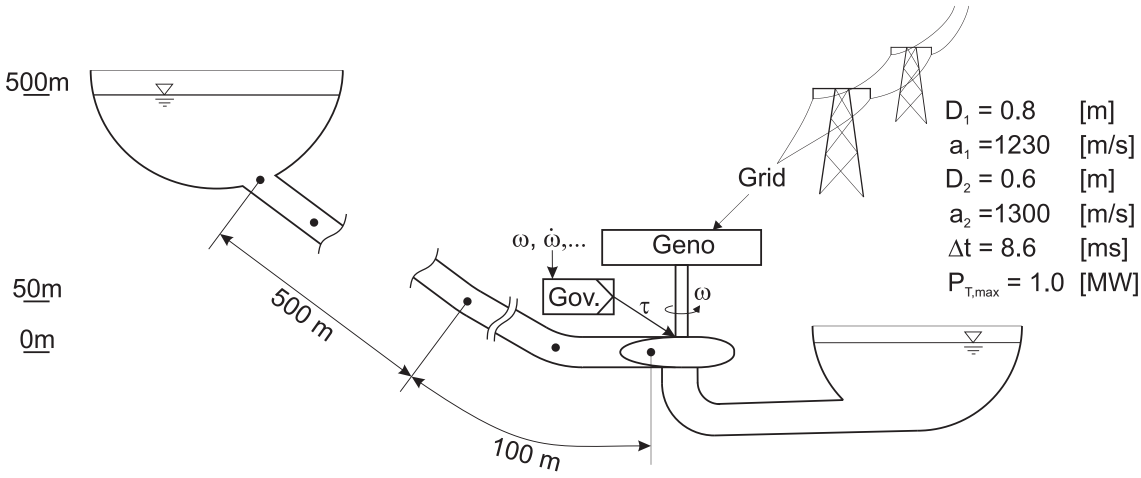

Figure 11: *Model of power plant*

Fig. 11 shows the computational model of a power plant. In addition to the piping the behavior of the turbine governor and the electrical grid are simulated. Fig.12 and 13 present some results from two calculations performed on this model.

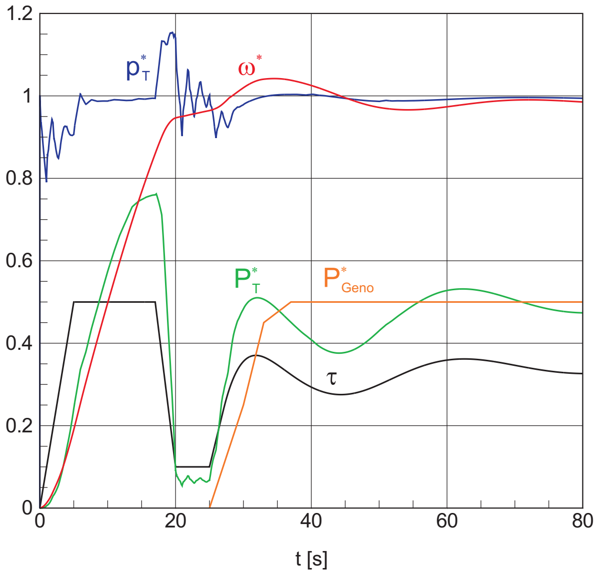

Figure 12: *Start up*

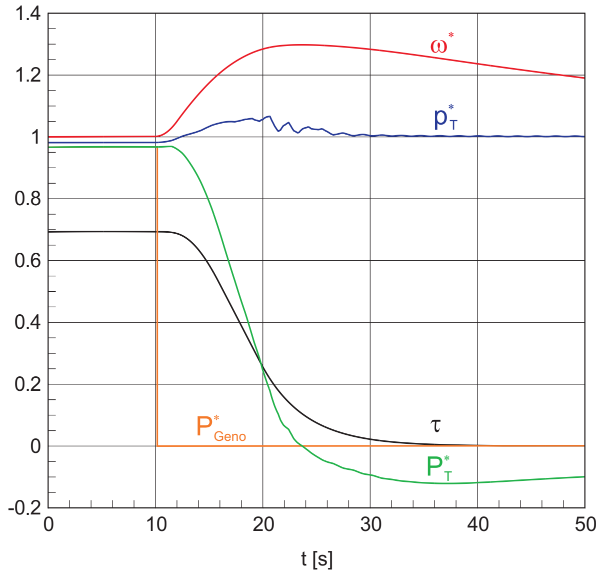

Figure 13: *Load rejection*

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