Author: Ulanov Alexander
E-MAIL: ulanov.alexander@gmail.com
ICQ NUMBER: 268865339
Skype name: ulanov.alexander

FEM study of the faults activation

1. Introduction

The present work considers coupled processes of elastic deformation and pore fluid diffusion in saturated porous media. The particularity of the problem is presence of an interior surface along which the parameters of the media may discontinue. The problem of this kind appears in various engineering applications. For example this approach can be used while subsidence of rocks under building foundation or sliding of bed near oil well. “Kols’kay” well geologic profile is presented on figure 1. Its depth is about 12 kilometers. As it’s easy to see, the surface under this field consists of various soils. Their positional relationship could influence on the form of the bore. Sliding of soil layers may cause clipping of bore hole. So the problem of predicting of soil motion is very important engineering application.

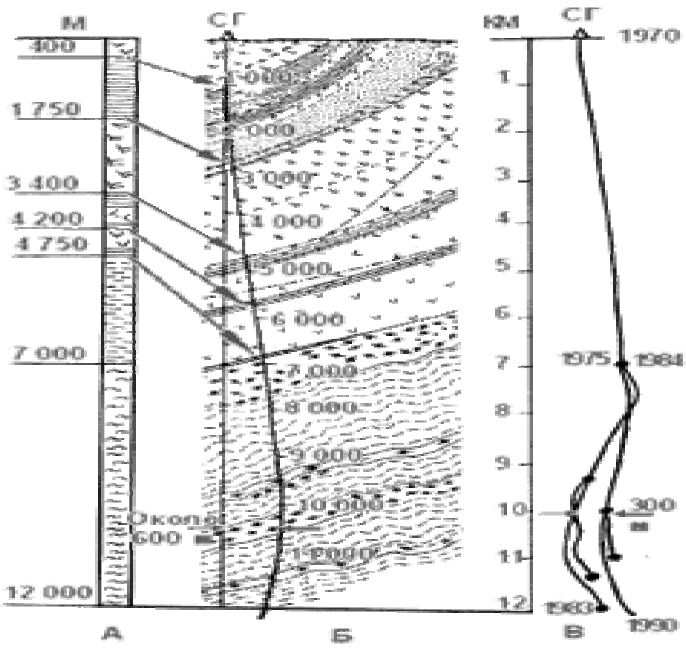

Figure 1: “Kols’kay” well geologic profile

The surface of media discontinue is referred as ‘interface surface’ or simply ‘interface’ in this text. We use in this work the interface element concept described by Goodman [1]. The interface area of infinitesimal thickness is introduced in the present work in variational problem formulation. And so the interface element appears naturally in the process of finite element discretization. Interface element properties are determined by interface conditions. Sufficient normal forces are supposed to appear in the contact layer, which provide normal displacement continuity. It is ensured by choice of large values of the elastic stiffness of the interface element. This stiffness may be interpreted from mathematical point of view as the penalty parameter. The forces appearing along interface area are limited by the Mohr-Coulomb law. It may lead to the slipping effect. As the result the problem becomes a nonlinear one. Besides elastic properties described above the interface layer is attributed with significant permeability, which may be interpreted as the penalty parameter for pore pressure as well. This property ensures pressure continuity on interface. For numerical solution of this problem in-house software based on Block Thomas' algorithm was developed.

2. Mathematical model

2.1 Governing equations

The equations describing coupled processes of elastic deformation and pore fluid diffusion in fluid-infiltrated elastic solids are as follows [2]:
The continuity equation:

$$M\frac{\partial p}{\partial t}-\frac{k}{\mu}\Delta p=-b\frac{\partial}{\partial t}div\mathbf{u} \tag{2.1}$$

The equilibrium equation:

$$G\Delta\mathbf{u}+\gamma\nabla div\mathbf{u}=b\nabla p \tag{2.2}$$

where p – the pore pressure of fluid, **u** – the displacement vector of the solid skeleton. The above equations contain following parameters: k is the coefficient of permeability, μ is the viscosity of the pore fluid, G is the shear modulus, *b* is the Biot's constant. Values M and γ are defined by expressions

$$M=\frac{b^2}{K_u-K}, \quad \gamma=K+\frac{G}{3} \tag{2.3}$$

$$b=1-\frac{K}{K_s} \tag{2.4}$$

where *K* is the bulk modulus of the overall skeleton (the drained bulk modulus), *Ku* is the bulk modulus under undrained conditions (the undrained bulk modulus). The Biot's constant may by written as (2.4), where *Ks* is the averaged bulk modulus of the solid grains. Note that $0\le b\le 1$ and *b* will be near its upper limit for soil-like materials, since then *K*<< *Ks*.

2.2 Variational formulation of the problem.

Let Ω be a domain in R^m (m=2 or 3), S is its boundary, and n is external normal to S. Suppose that the following formulas are valid for vectors u and q, defined in $\overline{\Omega}$ together with their first derivatives.

$$\int_\Omega \Delta\mathbf{u}\cdot\mathbf{q}d\Omega=-\int_\Omega(\nabla\mathbf{u})^T:\nabla\mathbf{q}d\Omega+\oint_S\frac{\partial\mathbf{u}}{\partial n}\cdot\mathbf{q}ds \tag{2.5}$$

$$\int_\Omega \nabla f\cdot\mathbf{q}d\Omega=-\int_\Omega f div\mathbf{q}d\Omega+\oint_S f\mathbf{q}\cdot\mathbf{n}ds \tag{2.6}$$

Multiply (2.2) scalarwise by vector-function q of the same class as u and integrate on domain Ω with boundary S using introduced formulas and grouping the integrals:

$$\int_{\Omega}\left(G(\nabla\mathbf{u})^{T} : \nabla\mathbf{q} + \gamma div\mathbf{u} div\mathbf{q} - bp div\mathbf{q}\right)d\Omega - \oint_{S}\left(G(\nabla\mathbf{u})^{T} + (\gamma div\mathbf{u} - bp)\mathbf{I}\right)\mathbf{n}\cdot\mathbf{q}ds = 0 \tag{2.7}$$

Designate $$\mathbf{F} = G(\nabla\mathbf{u})^{T} + (\gamma div\mathbf{u} - bp)\mathbf{I}, \quad \mathbf{F}_n = \mathbf{F}\mathbf{n}\,. \tag{2.8}$$

It is a stress acting normally to boundary S. The first summand herein is elastic stress, the second one is the stress occurring due to material volume change and the third summand is pore pressure of filling fluid. The stress occurring due to material volume change acts uniformly in all directions like pressure.

Inserting (2.8) into (2.7) write finally

$$\int_{\Omega}\left(G(\nabla\mathbf{u})^{T} : \nabla\mathbf{q} + \gamma div\mathbf{u} div\mathbf{q} - bp div\mathbf{q}\right)d\Omega - \oint_{S}\mathbf{F}_n\cdot\mathbf{q}ds = 0 \tag{2.9}$$

Using the similar approach, one can get variational formulation of continuity equation:

$$M\frac{\partial}{\partial t}\int_{\Omega} pq d\Omega - \int_{\Omega}\left(-\frac{k}{\mu}\nabla p + b\frac{\partial\mathbf{u}}{\partial t}\right)\cdot\nabla q d\Omega + \oint_{S}\left(-\frac{k}{\mu}\frac{\partial p}{\partial n} + b\frac{\partial\mathbf{u}}{\partial t}\cdot\mathbf{n}\right)q ds = 0 \tag{2.10}$$

Designate $$Q_n = \left(-\frac{k}{\mu}\nabla p + b\frac{\partial\mathbf{u}}{\partial t}\right)\cdot\mathbf{n} \tag{2.11}$$

This is a flux of fluid through boundary S directed normally to this boundary. The first term expresses the flow caused by a pressure difference, and the second one is fluid transfer together with a medium displacement (convective component). Inserting (2.11) into (2.10) we can write finally:

$$M\frac{\partial}{\partial t}\int_{\Omega} pq d\Omega - \int_{\Omega}\left(-\frac{k}{\mu}\nabla p + b\frac{\partial\mathbf{u}}{\partial t}\right)\cdot\nabla q d\Omega + \oint_{S} Q_n q ds = 0 \tag{2.12}$$

2.3 Interface model

Let two domains be Ω1 with a boundary S1 and Ω2 with a boundary S2, adjoining in an area Sc referred as the interface area (fig. 2). Remaining (external) boundary is $S = S_1 \bigcup S_2 \setminus S_c$. Designate external normals to areas Ω1 and Ω2 as **n**1 и **n**2. In the interface boundary **n**1 = – **n**2 = **n**. Values of quantities will be marked with superscript "+" if they are computed from side of normal n positive direction (i.e. from side of area Ω2), or with the superscript "–" if they are computed from side of normal n negative direction (i.e. from the side of area Ω1).

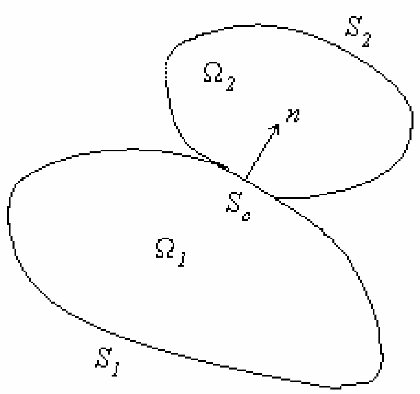

Figure 2: Interface model

Write equation (2.9) separately for each domain and sum up obtained equalities.

$$\int_{\Omega}\left(G(\nabla\mathbf{u})^{T}:\nabla\mathbf{q}+\gamma div\mathbf{u}div\mathbf{q}-bpdiv\mathbf{q}\right)d\Omega-\int_{S}\mathbf{F}_{n}\cdot\mathbf{q}ds-\int_{S_c}\left(\mathbf{F}_{n}^{-}\cdot\mathbf{q}^{-}-\mathbf{F}_{n}^{+}\cdot\mathbf{q}^{+}\right)ds=0\,. \quad (2.13)$$

Suppose that the interface is an infinitely thin flat layer with stiffness C and porosity D.
Force acting from the side of interface area on body Ω_2 (in direction $\mathbf{n}_1$)

$$\mathbf{F}_{n_1}=\mathbf{F}_1\mathbf{n}_1=-\mathbf{C}(\mathbf{u}^{+}-\mathbf{u}^{-}) \quad (2.14)$$

Force acting from the side of interface area on body Ω_1 (in direction $\mathbf{n}_2$)

$$\mathbf{F}_{n_2}=\mathbf{F}_2\mathbf{n}_2=-\mathbf{C}(\mathbf{u}^{-}-\mathbf{u}^{+}) \quad (2.15)$$

Turn with this expressions to normal **n**. Since $\mathbf{n}_1 = \mathbf{n}$, and $\mathbf{n}_2 = -\mathbf{n}$

$$\mathbf{F}_{n}^{+}=\mathbf{F}_1\mathbf{n}=-\mathbf{C}(\mathbf{u}^{+}-\mathbf{u}^{-})\,,\qquad \mathbf{F}_{n}^{-}=\mathbf{F}_2\mathbf{n}=-\mathbf{C}(\mathbf{u}^{+}-\mathbf{u}^{-})\,. \quad (2.16)$$

Note that normal forces appeared to be equal as it should be.

Inserting (2.16) into (2.13) we get equation:

$$\int_{\Omega}\left(G(\nabla\mathbf{u})^{T}:\nabla\mathbf{q}+\gamma div\mathbf{u}div\mathbf{q}-bpdiv\mathbf{q}\right)d\Omega-\int_{S}\mathbf{F}_{n}\cdot\mathbf{q}ds-\int_{S_c}\mathbf{C}(\mathbf{u}^{+}-\mathbf{u}^{-})\cdot(\mathbf{q}^{+}-\mathbf{q}^{-})ds=0\,. \quad (2.17)$$

Write equation (2.12) separately for each domain and sum up obtained equalities. We get

$$M\frac{\partial}{\partial t}\int_{\Omega}pqd\Omega-\int_{\Omega}\left(-\frac{k}{\mu}\nabla p+b\frac{\partial\mathbf{u}}{\partial t}\right)\cdot\nabla qd\Omega+\int_{S}Q_{n}qds+\int_{S_c}\left(Q_{n}^{-}q^{-}-Q_{n}^{+}q^{+}\right)ds=0 \quad (2.18)$$

Flux from contact area into Ω_2 (in $\mathbf{n}_1$ direction)

$$Q_{n_1}=-D(p^{-}-p^{+}) \quad (2.19)$$

and flux from contact area into Ω_1 (in $\mathbf{n}_2$ direction)

$$Q_{n_2}=-D(p^{+}-p^{-}) \quad (2.20)$$

Turning to normal **n** we get

$$Q_{n}^{-}=Q_{n}^{+}=D(p^{+}-p^{-}) \quad (2.21)$$

Thus the fluxes through the boundary are equal and the pressures are different. Pressure continuity is achieved by choice of sufficient high value of *D*.
Inserting flow expressions into (2.17) one can get:

$$M\frac{\partial}{\partial t}\int_{\Omega}pqd\Omega-\int_{\Omega}\left(-\frac{k}{\mu}\nabla p+b\frac{\partial\mathbf{u}}{\partial t}\right)\cdot\nabla qd\Omega+\int_{S}Q_{n}qds-\int_{S_c}D(p^{+}-p^{-})(q^{+}-q^{-})ds=0\,. \quad (2.22)$$

Equations (2.17) and (2.22) are basic ones for application of finite elements method (FEM).

2.4 Slip computation

Displacement and pressure equations obtained earlier contain terms taking into account relation of displacements and pressure on different interface sides of the domain Ω. Note difference in properties of possible displacement and pressure discontinuities on the interface. Pressure discontinuity on the interface occurs in the result of supposed finite permeability *D* of the interface elements at their zero thickness. It depends on fluid flow through the interface and is completely defined in the result of linear problem solution if the displacement equations are linear as well. Displacement discontinuity has other characteristics on the supposed interface which appears in this case as a plane of possible relative slide of the volume parts. Complexity of these characteristics is caused by strong nonlinearity of Coulomb – Mohr law leading to necessity of iteration process creation for solution of the problem of displacement and pressure field determination. In the considered case stiffness matrix has the following form:

$$\mathbf{C} = \begin{pmatrix} C_s & 0 & 0 \\ 0 & C_s & 0 \\ 0 & 0 & C_n \end{pmatrix} \qquad (2.23)$$

Sliding effect is supposed corresponding to Coulomb – Mohr law that the relative slide of two ground layers takes place in a point of the plane of possible slide when the following term is satisfied

$$|\sigma_S| \geq K\,|\sigma_n| + C_H \qquad (2.24)$$

Here σ_n is normal stress in the interface, σ_S is tangential stress in the interface, K is friction coefficient, C_H is cohesion stress. Let two ground layers be connected by an interface element having zero thickness but elastic properties. Suppose further that the ground located on both sides of the interface and the interface element can not shift relating each other in common nodes. Compute the stressed state of the system exposed to actions of predetermined forces supposing that the interface element and the ground have linear elastic properties. This problem is solved by the finite elements method.

Contact element properties are changing in the case of sliding. We put zero values for tangential components of stiffness matrix $C_s = 0$. So points on the top and on the bottom of the contact layer move independently. According to considered technique we define 3 basic steps of slip computation:

- Find deflected mode of considered body.
- Examine obtained solution for sliding (with reference to Mohr- Coulomb law).
- Following calculations using new form of stiffness matrix (in a case of sliding).

The second and the third step would repeat until sliding is possible.

2.5 Effect of nonuniform permeability

Nonuniform permeability effect can appear as a result of destruction of rock in contact layer. In case of sliding the last component of (2.18) will separate into 2 summands, where the second one is corresponding to the effect of nonuniform permeability:

$$\int_{\Omega_c} \left(Q_n^- q^- - Q_n^+ q^+\right) dx = \int_{\Omega_c} D_n \left(p^+ - p^-\right)\left(q^+ - q^-\right) d\Omega + \frac{1}{4\mu} \int_{\Omega_{sl}} D_s \nabla_{xy}\left(p^+ + p^-\right) \cdot \nabla_{xy}\left(q^+ + q^-\right) d\Omega \qquad (2.25)$$

$S_{sl} = [A_{sl}, B_{sl}]$ – sliding area.

3. Results

Numerical simulations of various geotechnical processes where obtained with developed in-house software. In some simulations results of our own code were compared with ANSYS. But in general cases commercial programs can’t produce geotechnical calculations. Fore example effect of nonuniform permeability is not included in some computational programs.

So as to illustrate the effect of nonuniform permeability let’s consider the establishment of linear pore pressure distribution with following conditions setting:

- The considered body is a cube (fig. 3) with horizontal contact layer.
- Designate initial conditions for pore pressure inside domain to be equal to zero.
- Designate values of pore pressure on lateral sides to be fixed.

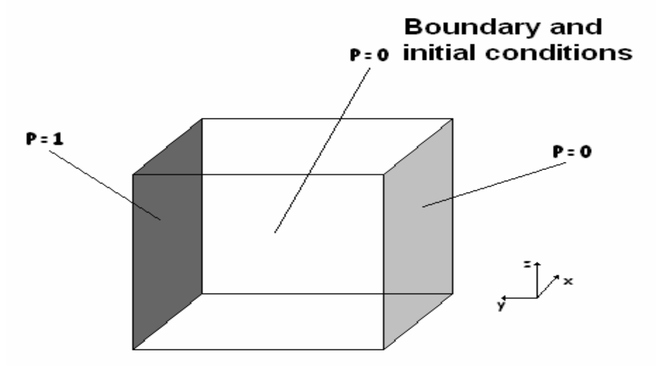

Figure 3: Boundary and initial conditions

Sliding of contact layer elements is caused by presence of the external load (fig. 4). Asymmetry distribution of sliding elements (fig. 4) is caused by unequal pore pressure values on the lateral sides. The gradient of pressure appeared during the process of establishment of linear distribution of pressure. From mathematical point of view this gradient plays role of additional force which is causing asymmetry distribution of sliding elements over the contact layer (fig. 4).

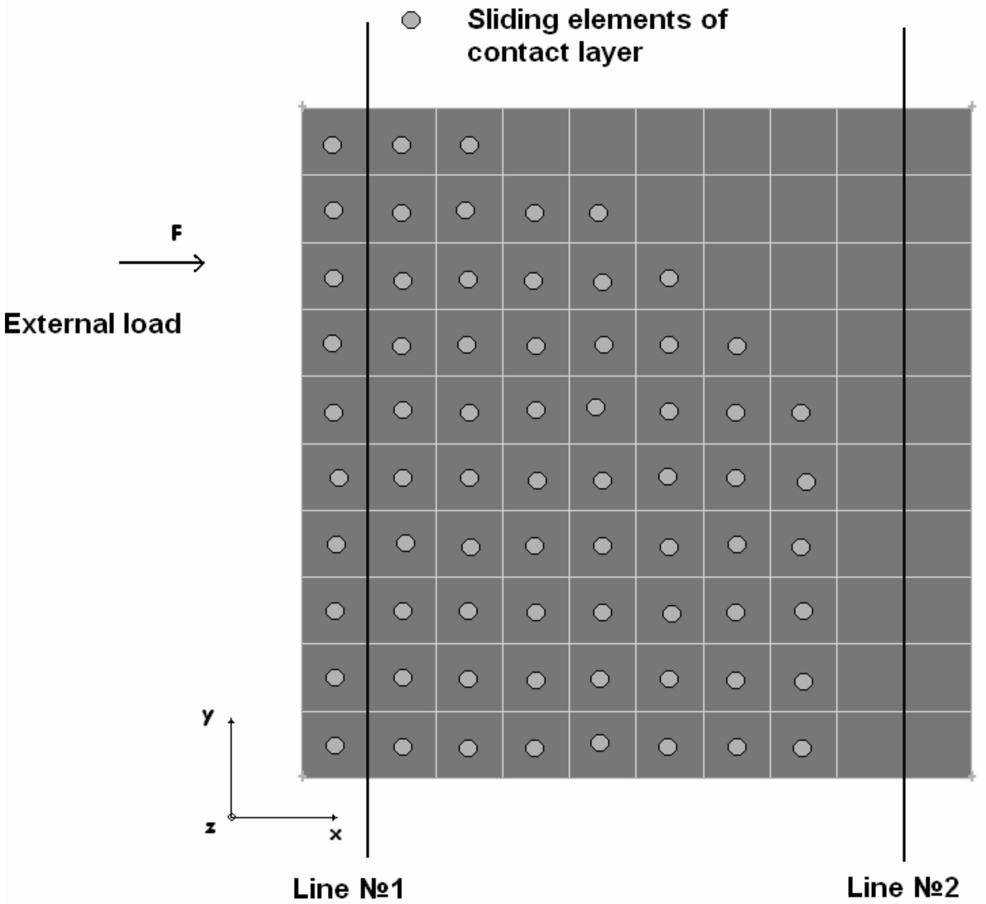

Figure 4: View from above on contact layer elements.

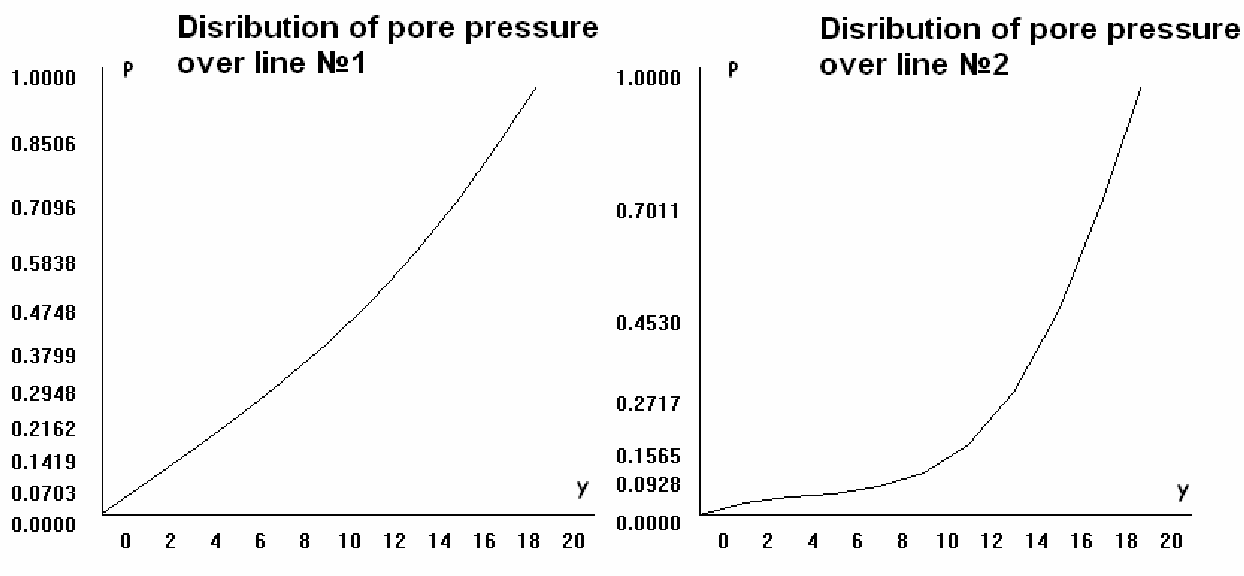

Figure 5: Distribution of pore pressure over lines 1 and 2.

Line number 1 is corresponding to sliding area, and line number 2 - to area where contact elements are still sticking. It's easy to see that establishment of pore pressure occurs more quickly in area where sliding elements appeared (left graph on fig. 5). This happens because of effect of nouniform (additional) permeability.

4. Conclusions and further researches

At the moment various geotechnical phenomena where included in our own code. The developed mathematical model successfully realizes geomechanical processes like pore pressure distribution, nonuniform permeability and sliding in contact layer of course. The work is ongoing. As the next step of this research we propose developing of parallel version of this software.

5. References

[1] Zienkievicz O. C., Best B., Dullage C., Stagg K. Analysis of nonlinear problems in rock mechanics with particular reference to jointed rock systems. Proceedings of the Second International Congress on Rock Mechanics, Belgrade, 1970.

[2] Goodman R. E., Taylor R. L., Brekke T. L. A model for the mechanics of jointed rock. Journal of the Soil Mechanics and Foudations Division, Proceedings of American Society of Civil Engineers SM 10 (ASCE) 1968; 94:638-659.