Course 5: Mechatronics - Foundations and Applications

Snakelike robots locomotions control

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May 29, 2006

Abstract

In this paper you can find a review of snakelike robot constructions as well as report on biometrical requisites for development such devices and give a some description of mechanical and mathematical models snakes movement. Point out on existence rational model of snakes's movement. This model is very helpful to develop effective algorithms to control locomotion snakelike robots witch have no wheels. In report described the real hardware snakelike robot locomotion control and give a detailed description of snakelike robot developed in Scientific Research and Design Institute of Robotics and Technical Cybernetics in Saint Petersburg, Russia. This model have already performed showing different modes of locomotion.

Contents

1 Introduction

Nowadays snakelike robots are an actively upcoming area in robotics. Frequently the term snake like robot is applied to all hyper redundant robots; those are consisted from modules connected by active or passive joints. Among these devices the real snakelike robots are those that use the wave movements of the successive module chains for locomotion on hard surfaces and in liquids. At most attention of specialist's serpentine designs is explained by their desire to use the unique possibilities of the limbless to adapt to surfaces and environments. These possibilities are relatively harmonize with the simple anatomy.

The initial developments of the snakelike robots were executed in 70-s by Hirose group of investigators. He made the analysis of limblesses motions experimental data and suggested mathematical description of the snake's instant form. The curve was called "serpenoid" and is used for the snakelike robot's control assignment. The first designs of Hirose snakelike robots had modules with small passive wheels. The same design was used in projects of Gavin Miller. Robot that has no wheeled supports for its motion is more close to the biological analogs. The first successful developments of this type of robot appeared at the end of the 90-s at Carnegie Mellon University. At the same time active development of algorithms of snakelike robot locomotion control was carried on. In the articles by Ostrovsky, Chirikjian, Choset, Dowling [1, 2, 3] and other scientists various decisions for control generating that could provide some locomotion modes in the snakelike robots have been rendered.

It is necessary to point out that for a long time a rational model of the snake's movement has not been presented. Such model would be very helpful to develop effective algorithms to control movements of the snakelike robots that have no wheels. One of quoted movement models has been offered in 1970. by A.I. Dobrolyubov. The mechanical moving model of running waves of the deformation having local motionless contact, has allowed to give qualitatively true description of factors influencing on moving of a flexible body with use various locomotion modes. Unfortunately, it did not contain the proved mathematical description.

In 2002-2004 A.A. Ivanov suggested some mathematical models of the flexible body dynamics and kinematics [4]. These mathematical models allow to describe experimental data on snakes' kinematics within the limits of biological experiment precision. Having mathematical models let us make right constructive decisions not only on mechanical structure of the snakelike robot but also on the purposeful movement system control with the use of different modes of locomotion.

2 Why snakelike robots?

2.1 Advantages

Recent years man's sphere of activity has spread in to various directions due to development of technology. In many cases, such activity is accompanied with danger, then necessity for work done by robot is increasing. Furthermore, the multi functional robot which cam do many operations is desirable. There-fore, development of a robot which has redundancy is expected. Snakelike robots are a typical example of a robot with redundant degree of freedom. Furthermore, they can move both in a narrow place and in place with a height difference. Moreover, since it consists of many joints, it is expectable to be multi-functional. A lot of research into the object for disaster relief or dangerous zone work is made. The feature of redundancy in snakelike robots is used.

Only in the last few decades researchers and designers began to replicate the general movements of animals in mechanisms. The general motivation for serpentine locomotors construction are environments where traditional machines are precluded due to their size or shape and where appendages such as wheels or legs cause entrapment or failure. Example environments include tight spaces, long narrow interior traverses, and travel over loose materials and terrains. Serpentine mechanisms hold particular fascination due to the singular motions usually associated with animals such as snakes and tentacles. A few terrestrial mobile devices move without use of wheels or legs; those that exist in the laboratory have exhibited only the rough features of natural limbless locomotors such as snakes. Serpentine features include serial chains of actuators capable of subtending small curvatures. However many of these prior efforts incorporated non-biological features: use of casters for support and propulsion or use of fixed pins for support and traction. Other broad features of these prior robots include use of models that explicitly describe the shape of the robot, the use of tensor mechanisms that limit curvatures and forms and mechanism designs that are useless for application. There are significant challenges in designing, building and controlling practical limbless mechanisms that are capable of locomotion without traditional forms of propulsion and actuation. These challenges include configuration, design and geometry of the form, determining the number and arrangement of actuators, routing power and signal distribution, and robot control. Wheels offer smooth and efficient locomotion but often require modifications to terrains for best use; even all-wheel-drive mechanisms are limited in type and scale of terrain that can be traversed. Serpentine locomotors possess a number of potential advantages beyond the capabilities of most wheeled and legged vehicles.

2.2 Applications

The snakelike robots have a number of advantages like stability, terrainability (is the ability of a vehicle to traverse rough terrain), traction (is the force that can be applied to propel a vehicle.) , redundancy and some others, not so important for mobile robots.

But in spite of many advantages the snakelike robots can't be use everywhere. It is because of many problems, which researchers can't solve properly yet. The snakelike robots are very difficult to design, build and control.

The researchers are working to solve all these problems from different points of view. And they propose many applications. The Japanese researchers made their robots for examination blockages after earthquake for search of survived people. American researchers build snakelike robots for planet's exploration. In the same way researchers propose medical applications, examination hard-to-reach areas - NASA, Security Organizations, the firms working with tube inspection and of course military department are interested in these applications.

3 Biomechanics of snake

3.1 Bionics

Bionics is the application of methods and systems found in nature to the study and design of engineering systems and modern technology. Snakes are the ultimate example of limbless animals; the modes and quality of their locomotion exceeds all other biological limbless locomotors. The snake is a vertebrate, an animal with a backbone, and has the largest number of vertebrae of any animal: between 100-400 vertebrae, depending on the species. Snake vertebral articulation is one of the most complex of all vertebrates.

Snakes and other limbless animals have been objects of study for centuries. However, until recently, little research has focused on the detailed mechanics of serpentine locomotion. Yet, there is a fair amount of information on the qualitative aspects of snake locomotion. There are several broad classes of limbless locomotion; these include lateral undulation, concertina, sidewinding, rectilinear, slide-pushing and other less common forms. These classes are, in fact, gaits, a term normally associated with legged animals. Gaits are repetitive patterns

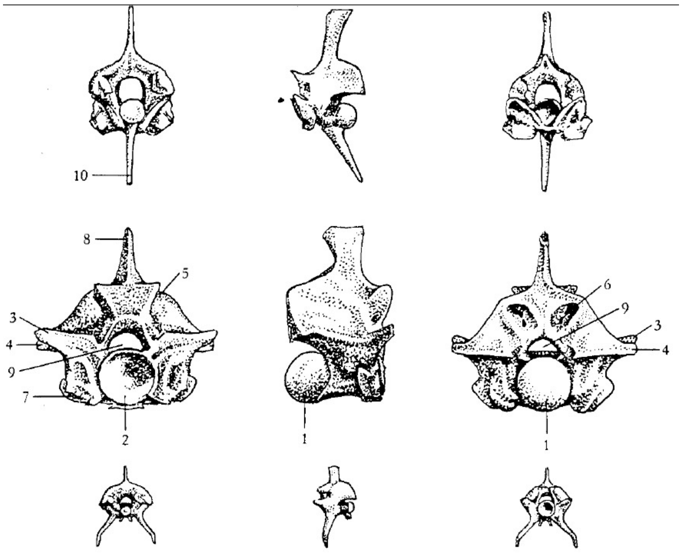


Figure 1: Snake vertebral

of movement used to change speed, adapt to terrain, and improve stability. Gaits are often chosen because they are more economical for a particular situation.

3.2 Locomotion modes

Lateral Undulation

Lateral undulation is the most frequently used form of snake locomotion for most snakes. All parts of the body move simultaneously, experiencing continuous sliding contact with the ground. It is a sliding motion with all parts moving at the same speed that occurs through the propagation of waves from the front to rear of the snake. The snake remains in contact with surface and the motion is similar to a swimming motion. Energy consumption is comparable to that of legged animals of similar scale. During lateral undulation, the snake pushes against features in the environment to facilitate forward movement.

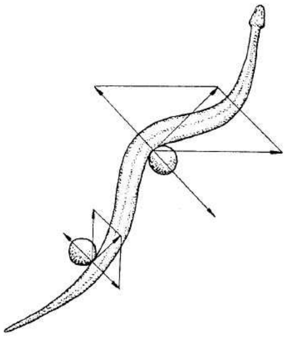


Figure 2: Lateral Undulation

Lateral undulation is the only form of biological snake locomotion that doesn't use static contacts between snake and substrate. The ideal path is a single track along which the snake slides. Lateral undulation requires a minimum of three contact points for continuous forward progress: two to generate force and the third to balance forces to move in a particular direction.

Lateral undulation is unsuited for smooth, low-friction surfaces and narrow corridors. Nor is it well suited for short stouter animals or for large heavy-bodied snakes because they are unable to either subtend the curves required or the body mass and environment tend to significantly reduce its efficacy. Both wheels and legs use static contacts for propulsion but lateral undulation in snakes offers an interesting variant using sliding or dynamic friction. This is not as inefficient as it might first appear. However, the complexity of snake anatomy may make it difficult to realize these advantages in mechanisms.

Concertina

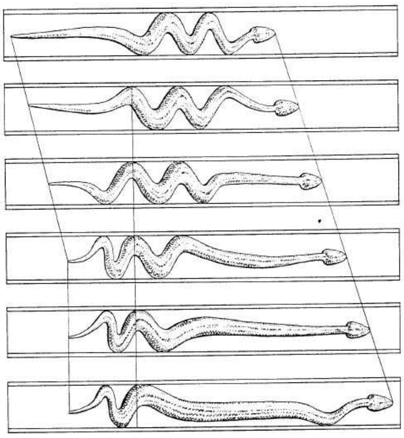


Figure 3: Concertina

The concertina gait derives its name from a small accordion-like instrument because of the shape and motion of the snake body. Concertina progression provides a base in which parts of the body stop for purchase and other parts move forward. The sequence repeats, and the snake moves forward. It is usually used in confined areas, such as tunnels, where the snake cannot utilize the full amplitude of other gaits. As shown in Figure 3, the trunk straightens forward of each contact site and is simultaneously set down in a curved pattern at the rearward end of each site. As a result the musculature needs to be activated at or near moving portions of the trunk. The key element of concertina locomotion is the utilization of the difference between high forces with the static coefficient of friction and low forces with the dynamic coefficient of friction along different parts of the body. Due to momentum changes, static friction, and slower speeds, concertina is a relatively inefficient mode of locomotion, but forms of concertina allow traverses not otherwise possible, such as moving along wires and cables as well as through tree branches. Concertina movement resembles, in some ways, the motion of worms; parts of the body remain in place and other parts move forward. It would also appear to be simpler, perhaps, to implement in a mechanism, than other forms of snake locomotion.

Sidewinding

Sidewinding is the use of continuous and alternating waves of lateral bending. A downward force is exerted for purchase on low shear surfaces like sand or loose soil; this mode establishes rolling static contacts to cross relatively smooth substrates. There are only two contact patches while the snake is in motion. The technique minimizes slippage and is even more efficient than lateral undulation. Some sidewinding snakes have been observed to travel kilometer-length

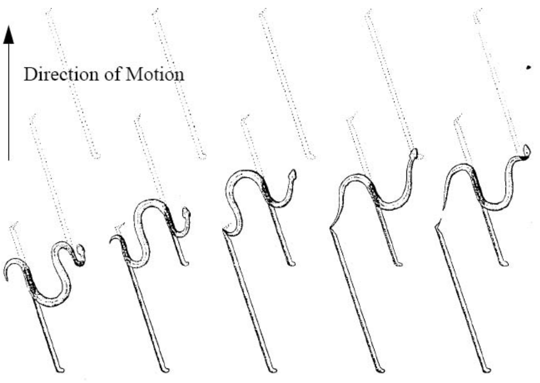


Figure 4: Sidewinding

distances continuously. Sidewinding is used primarily by snakes in desert regions where loose soils and sands are prevalent. The development of sidewinding may be related both to the need for traction on low shear surfaces such as sand and the need to avoid the high temperatures of desert terrain. As shown in Figure 2-4, sidewinding can be thought of as the 'peeling' of the body from one track to the next. The tracks, or lines, show the rolling of the body contacts during locomotion.

Rectilinear

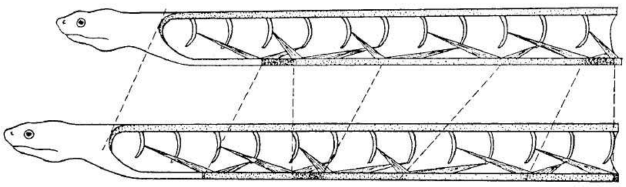


Figure 5: Rectilinear

Rectilinear progression uses movements of skin with respect to the skeleton to 'rachet' the body along the ground. Rectilinear motion is a slower, creeping motion using the belly to provide traction through anchoring and is typically used by larger snakes. Rectilinear motion was once conjectured to result from 'Rib-walking,' an active movement of the ribs. However, this was conclusively disproved in through x-ray observations of a snake in motion. Muscles connected from the ribs to the elastic skin provide the propulsive motions through reciprocating or racheting movements. In rectilinear locomotion, several portions of the body are in contact with the ground at any moment, and the gait uses symmetrical rather than staggered waves of contraction. A section of the skin of the belly is drawn forward so belly scales are bunched. This part of the body is then pressed down, and ventral edges engage the surface. Then the body slides forwards within the skin until it is in normal alignment with skin, and the motion repeats. Only small vertical motions are needed for rectilinear locomotion.

Other Snake Locomotion Modes

Other forms of limbless locomotion include slidepushing, saltation, burrowing and climbing. Slidepushing is a gait used in times of stress where anteriorly propagating waves move more quickly backwards than the snake moves forwards. A great deal of sliding and motion occur without a corresponding forward progression. Saltation is the jumping the near-vertical walls and trunks of trees. Some saltating snakes can leap gaps of a meter or more, sometimes vertically. This requires storage and release of a lot of energy and, additionally, involves a ballistic phase of motion during which control is difficult. Other extraordinary modes are used by certain asian tree snakes that glide through the air by opening the rib cage to form a gliding surface. The amazing thing about this mode of snake travel it is not how well it flies, but that it flies at all!

4 Review: mechanic model of snakelike robots

For correct designing and locomotion control are necessary to construct right mechanical model. Researchers applied various approaches to control and construction snakelike robots. One of the most known works in this area was by: Hirose, Chirikjian, Dowling, CONRO, Dobrolyubov, Ivanov. Other works which can be found, as a rule, directly or indirectly refer to these researchers who make an important contribution in this area of mechatronics.

4.1 Hirose

One of the most famous works was made by Hirose and Umetami, in the early 1970's, they were the first who explore, design and develop limbless locomotors [5, 6, 7, 8].

Hirose have two motives for beginning biomechanical research on the movement of snakes. The first motive was that, up until that time, the fundamental problem of "How is it that a snake can go forward without legs?" largely remained unanswered, and this required an engineering analysis. The second motive grew from the expectation that a "snake-like robot", which would be modeled on a snake would have a particularly broad functionality while maintaining a simple shape. The future possibilities of serpent robots can be anticipated from the fact, as indicated in fig 6, that the body of a snake, which has the simple form of a rope, functions as "legs" when moving, as "arms" when traversing branches, and as "fingers" when grasping something.

Figure 6:

When beginning this research, in order to explain the dynamics of the creeping propulsion movement of snakes on level ground, a basic motion equation for this was derived, and numerous running experiments were conducted using striped snakes. Photo. 2, is one example of this. The conditions for moving on level ground were investigated by rigging an electro-muscular meter and a normal force meter on the torso of the snake. From these experiments, it was found that:

1. The waveform that the snake assumes during creeping movement is a curve which changes sinusoidally along the curvature of the body, and Hirose made a formula for this, calling it a serpenoid curve. These equations are shown below:

$$x(s) = sJ_0(\alpha) + \frac{4l}{\pi}\sum_{m=1}\frac{(-1)^{2m}}{2m}J_{2m}(\alpha)\sin\left(m\pi\frac{s}{l}\right); \tag{1}$$

$$y(s) = \frac{4l}{\pi}\sum_{m=1}(-1)^{m-1}\frac{J_{2m-1}(\alpha)}{2m-1}\sin\left(\frac{2m-1}{2}\pi\frac{s}{l}\right); \tag{2}$$

2. The action by which one part of the body floats up during advancement, called sinus-lifting in Photo. 3, can be interpreted as an action which concentrates the body weight on the part that can most easily slip, and this functions to prevent slippage, as indicated in Fig. 6 and as in Fig. 7

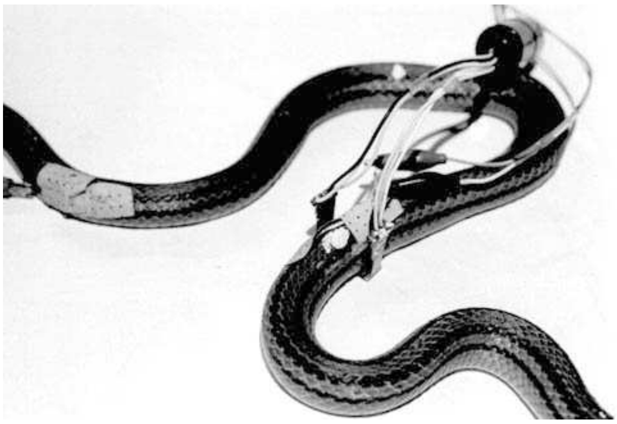


Figure 7:

3. A variety of positions can be considered for the propulsion motion in the corner part within the labyrinth, but the most appropriate body form is (d), and this was also experimentally verified as in Fig. 8

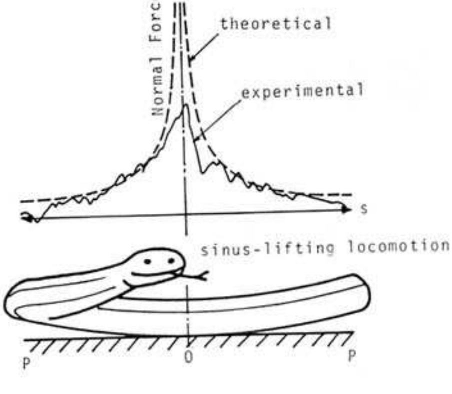


Figure 8:

Hirose has a sustaining interest in limbless locomotion and designed and built several robots over decades. He termed the devices Active Cord Mechanisms or ACM's. Hirose focused on developing robots that could perform lateral undulation and later developed a series of wheeled coupled-mobility devices that followed from this work.

Hirose's development of modeling and control first derived expressions of force and power as functions of distance and torque along the curve described by the snake. The curve was then derived and compared with results from natural snake locomotion. The curve, termed serpenoid, has curvatures that vary sinusoidally along the length of the body axis.

This curve is different from sinusoidal or even clothoid curves. Comparisons with natural snakes across constant friction surfaces showed close agreement between the serpenoid curve and the empirical data.

Hirose then went on to develop models for the distribution of muscular (actuator) forces along the body. This was done for normal and tangential forces as well as power distribution. Again, the developed models closely correlated to muscle exertion data and force measurements from natural snake movements.

The experiments were primarily of a uniform nature, but Hirose recognized that snakes quickly adapt locally to variations in terrain and environment. The next issue was to characterize this adaptation. From observation it was noticed that snake locomotion is not necessarily a two-dimensional problem; in fact during higher speed motions, snakes use ventral motions to actively distribute their weight to those areas where propulsion is maintained.

Further study developed relationships between amplitudes and wavelengths of the motion and local friction conditions, as well as morphological features of the snake such as vertebrae motion and muscles (actuators). Models for locomotion in rough terrain where obstacle contact is made were also developed and correlated with snake motions.

Hirose examined the construction of mechanisms that were able to perform lateral undulation. By calculating torques, velocities and power required, Hirose was able to provide design guidelines for the actuators and drive trains. The next development was a distributed control scheme wherein each link could respond independently. In Hirose's work the control took the form of angle commands at each joint. The variables were simply related closely to the amplitude, wavelength and velocity of the body axis. Steering of the robot was accomplished by biasing the control to adjust curvature in a section of the body.

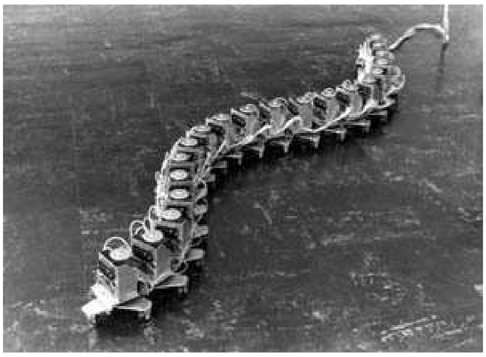


Figure 9:

A 20 link mechanism weighing 28kg was constructed. Link actuation was accomplished with DC motors coupled to a caster board and potentiometers were used for feedback. Later, after a motor change, the weight was reduced to 13kg. To accommodate unknown environments is tactile sensing required; this was the next step in Hirose's work. Small contact switches provided this information to the controller. As shown in Figure 2-10, this robot could negotiate and propel itself through winding tracks. The developments included a control technique called lateral inhibition tactile signal processing, which provided for contact and reflex motions. The shape of the body was varied according to the second derivative of the sensed contact pressure and responded appropriately to provide forward progress.

All of Hirose's locomotors used either powered wheels or passive casters and the only locomotion mode studied was lateral undulation. Hirose and his colleagues have gone on to develop an elastic elephant-like trunk, a large serpentine mechanism for interior inspection of turbines and small manipulators for surgical applications. Hirose's work in serpentine robots is probably the most complete of all work in this area. He dealt with issues of mechanism, control, sensing and modeling of natural animals. However, the mechanisms used wheels, the terrains for the ACM's were 2D only, and the mechanism used only lateral undulation as the locomotion mode. The configuration, while not practical for application use, was a great advance in serpentine robots.

Figure 10:

4.2 Burdick and Chirikjian

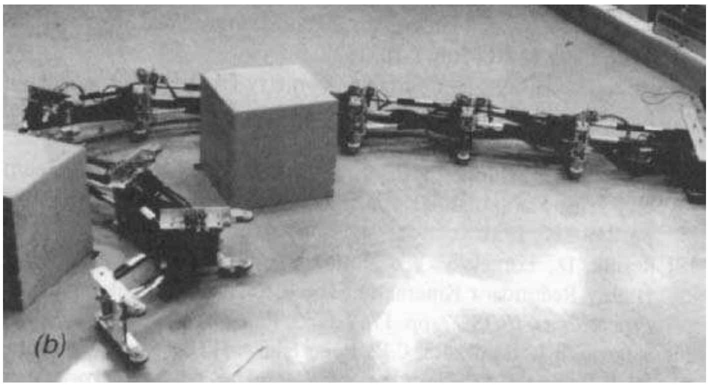


Figure 11:

Joel Burdick and his students at Caltech, have pursued work in serpentine manipulation and locomotion for several years. Chirikjian's thesis presented a framework for kinematics and motion planning of serpentine mechanisms. Curves in three dimensions, R3, are defined to provide a general means of parameterizing curves and sets of reference frames. In addition to describing the curve shapes, they extended features to allow roll distribution along the curve and extensions and contractions along curve segments. These were then used to specify serpentine configurations.

Since most manipulators do not describe continuous curves, there remained the problem of fitting rigid link devices to the desired curve. A general parallel algorithm was found for fitting manipulator segments to the desired curve. The modal approach was then used to resolve the 'excess' degrees of freedom in hyper-redundant robots to carry out specified tasks. The modal approach provides a means of characterizing the shape and motions without developing full inverse kinematics, which have an infinite number of solutions. A series of specified functions could be specified in modal form, and the problem became finding modal participation factors to satisfy, as best can be done, the task constraints. Optimal techniques for minimizing measures of bending, extension, etcetera, were then developed via the calculus of variations. One issue with these optimal techniques is the selection of cost functions to evaluate configurations. That is, how to determine the 'goodness' of a particular solution.

Chirikjian described obstacle avoidance using this set of tools and it was assumed that paths were provided through traditional motion planning techniques. An additional issue addressed is that of time, that is, velocity, for the solutions. A series of arcs and lines were used to create a path along which the manipulator sections can move. But, independent of the path formulations, the previous solutions to kinematics could fit manipulator configurations and trajectories.

Locomotion through sequences and patterns of geometries was developed next. The extensible locomotion modes were traveling wave, similar to rectilinear motion in snakes or caterpillars, and stationary wave, similar to inchworm motion where the advancing wave remains in the same position with respect to body coordinates.

The extensible modes are similar to earthworm locomotion where segments provide extension and contraction to propel the robot. To avoid the need for differential friction, portions of the body can be raised to facilitate this motion. Descriptions of techniques for non-flat floors are also developed. Intriguing ideas were also introduced using serpentine robots to provide grasping and manipulation capabilities. The mechanism could contact and wrap about an object; the propagation of a wave or extension of the links caused the object to move in a desired direction. These techniques could be used to simultaneously grasp, move and manipulate objects. A mechanism, a variable geometry truss configuration, was designed and built and is shown in Figure 2-11. The mechanism was comprised of commercial linear actuators; a simple modular and maintainable approach to design was used. A variety of tests using the methods described above were conducted and a number of successful experiments in control and locomotion were carried out.

Key to Chirikjian and Burdick's work was the modeling of the robot as a 3D shape and sequence of shapes. This enabled a variety of techniques in trajectory planning and path generation. The mechanism also allowed exploration of non-snake-like extensible gaits. The mechanism however was primarily a fixed base device and a couple of limited gaits were demonstrated on the robot. Additionally, rachet wheels were used in locomotion . Sidewinding was also formulated in piecewise continuous curves in and, although the exact shape of the body was not necessarily snake-like. the general form of the motion was identical to that of snakes.

Later work by Burdick with J. Ostrowski explored the use of geometric mechanics to formulate general notions of locomotion. Two systems were evaluated in this context: a 'snake-board' which is an actively articulated skate board, and Hirose's ACM [3]. Other related work at Caltech included the work on geometric phases to describe robot locomotion .

Choset also developed path planning methods for highly articulated robots such as snake robots. He developed the Generalized Voronoi Graph (GVG) and Hierarchical GVG for use in sensor-based planning motion schemes. The techniques utilize concise descriptions of the topological spaces to build paths. A key feature of the work is that is does not require a priori knowledge of the world.

4.3 Dowling

The second greatest contribution in development snakelike robot was made by K.Dowling in the middle of 1990 in CMU. His researches have provided for the following stage of snakelike robots. Development began to move not only in one plane and use, only lateral undulation. The design of his robot was simple, it represented a chain of one-sedate modules, but axes of modules were orthogonal relatively to each other in a circuit. Application of such design was the most similar to real snakes. K. Dowling has carried out researches not only in the field of design, but also in the field of control. K. Dowling has selected specific resistance as the measure of performance for learning locomotion. It provides a notion both of energy, time and weight of the robot. It utilizes two measurements that can be found from both the physical model and from simulation. It is easily and quickly calculated and provides a clean and understandable metric for evaluation during the learning process [1].

Whatever the particular metric value, it is not a good idea to draw too many conclusions or provide close comparisons to other robots. It's too easy to contrive a metric that favors a particular robot. It's also too easy to draw conclusions about vehicles that don't take environment and task into account. However, it is important to realize that metrics reveal only how well a vehicle did on a particular performance measure. It does not reveal why, although it can provide clues, and, finally, it does not directly reveal how to make the performance better. It can be used as a tool to ascribe trends through the use of small changes in the control techniques and hence, develop a better understanding of what makes a better gait. The metric developed is used as part of the learning process and placed into the overall framework to teach the snake robot to locomote. Machine learning techniques evaluate past data to form insights on future performance; learning provides improved performance through experience. Learning and Optimization examines learning locomotion for simulated mechanisms and actual robots as well as criteria and structures for learning.

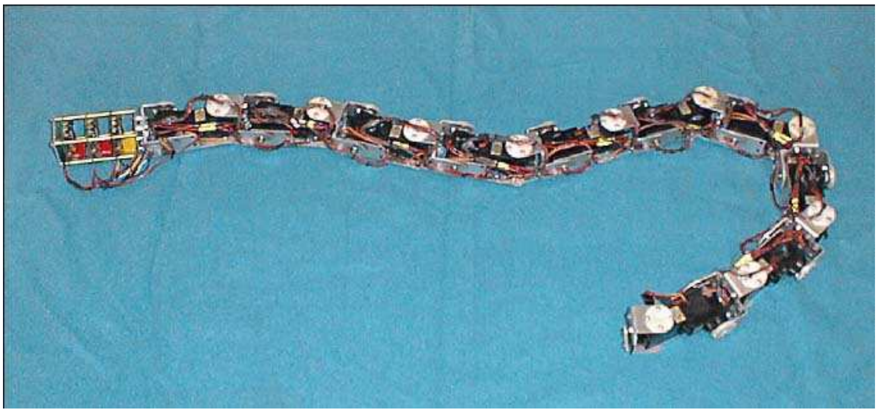


Figure 12:

4.4 Howie Choset - CMU

Choset's research group in Biorobotics Lab of CMU has constructed many snake robots named SnakeBots Design which is applied by these researchers one of the most widespread recently. Up to them it used Hirose in robot ACM-R3. Orthogonal connection of hinges with one degree of freedom. Such connection allows to come nearer to a true backbone of the snake at reduction of distance between axes of the next hinges. Recently Biorobotics lab has designed the order 8 snakelike robots. This laboratory solves 2 primary problems: improvement of a design and planning of movement as application of snakelike robots in city conditions for search and rescue is planned. Therefore robots should maneuver in three measurements. The second application for which snakelike robots are under construction are a application in medicine for minimally aggressive surgery to not do big sections in a body of the patient. Jobs above the robot for cardiovascular surgery are now conducted.

Figure 13:

Once the snake robot is built, it still requires control. Simple engineering hacks alone are not sufficient to coordinate the internal degrees of freedom to allow for purposeful motion. Essentially, the robot must plan in a multi-dimensional, one for each degree-of-freedom, space. Choset's approach uses a topological map of the space, which reduces planning from a multidimensional search problem to a one-dimensional search. In 1997, Choset received an NSF Career award to develop a topological map based on a retract-like structure. However, the retract-like structure is not enough; each path generated by the retract must be optimized so that the snake robot can more easily follow it. Naturally, with all optimization problems, we must contend with local minima. Here, they take recourse to homotopy theory where the retract-like structure seeds a set of candidate searches of the robot's free space, one of which leads to the global optimum. Essentially, they are exploiting the natural topology encoded in the free space to divide it into regions each having simple structure and optimizing within each simple space a cost function. This approach is general: the cost function can be anything: path length, safety, energy, etc. For snake robots, we have defined a "snake robot" cost function.

4.5 Dobrolyubov A.I.

The main conclusion - which can be made of Dobrolyubov's works - "genetic relationship" wheels and waves and Snakes are using rolling motion [9].

Anybody from above listed researchers did not speak about the mechanic of movement of the snake. And for this reason anybody of them cannot correctly construct correct model of management. Therefore using the formula which describes a curve serpenoid select parameters of this curve by means of various methods: evolutionary algorithms, neuronet, plastic networks, etc. The first person who has tried to formulate a rule of self-movement on a firm surface of coiled essences there was Dobrolyubov A.I. At each moment of time a moving body should have even one motionless point which is based a support. It from the third law of Newton the law of dry friction.

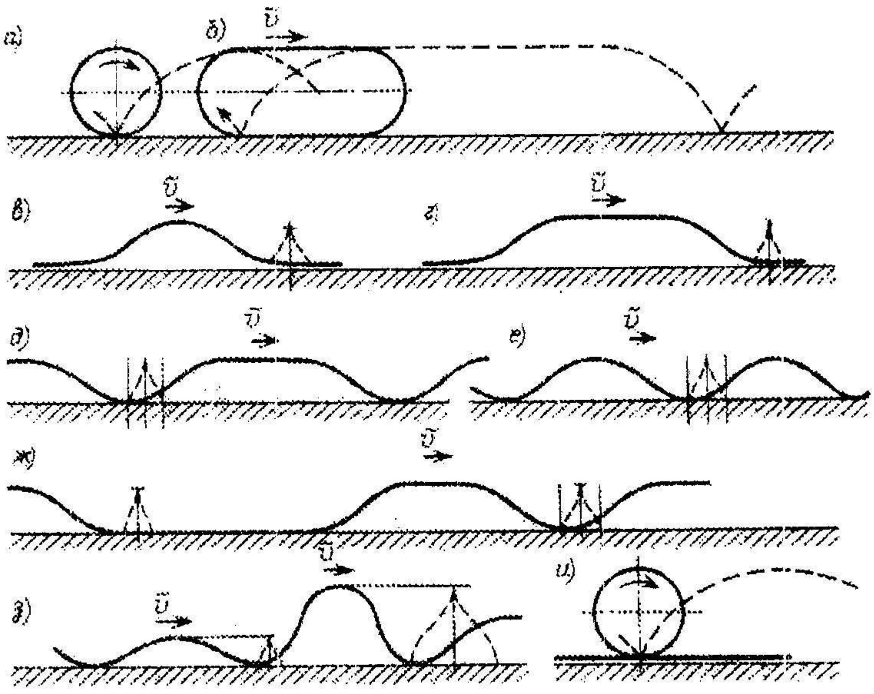


Figure 14:

Some points of a moving body or set of bodies during movement should vary periodically roles: mobile points become motionless and on the contrary. On character of this procedure of locomotion can be divided into two big classes: pacing when reference points of a body only during some moments of time pass from motionless in a mobile condition and back, and rolling when these transitions are carried out continuously. Snakes can move by pacing and rolling. Carry of points of a support of the essences, moving in the way rolling, can be various. (see pictures)

The main conclusion - which can be made of Dobrolyubov's works - " genetic relationship " wheels and waves. And snakes are using rolling motion.

4.6 Ivanov A.A.

Unfortunately, despite the enormous experimental data get as a result of supervision and registration of kinetic characteristics of snakes, the rational model of movement of the snake has not been formulated [4].

All existing models of movement of the snake are attached to separate kinds of its (modes) locomotion and base on the assumption of fantastic ability of the snake to adapt to external force factors and to raise various groups of muscles depending on size and directions of dynamic and static forces of dry friction. Models of the snake robot movement are under construction with the use of the optimum control device that is traditional for a robotics. For system of the firm bodies connected by hinges, the set of the operating generalized forces in the drives providing performance of one or several criteria and restrictions sorted out. Models are limited to consideration of flat movement of chain systems of bodies, and laws of control are found in view of the friction forces balance analyses distributed along "body" of the robot and led to the points of bodies, make the model .

Reptile movement - essentially not flat. Ivanov A.A. has offered a continuous spatial model of snake or snakelike robot kinematics within the limits of which all known modes of snake locomotion can be performed. Ivanov A.A. initially decided to constructs of model of biomorphous robot, which has no wheels and at moving base directly on models' cases. Absolutely authentic factor is presence of cross-section running waves along the snake's body. Also it is known, as it shown above, that the running waves of deformation having contact points to a motionless surface, can move along it. From the kinematics point of view moving on the surface of contact of the running wave along the flexible body - Is the obvious fact that also can be explained by simple examples. We shall consider moving of various deformations along a final length tape (a material piece), laying on the rough surface.

1. Passage of a single cross-section smooth impulse on a piece having the initial rectilinear form, leads to moving of the piece relatively to motionless system of coordinates in the same direction on the distance equal to a difference of curve length, forming an impulse, and sizes of its projection to a motionless "basic" plane (see a Fig. 15). We'll note that at each moment (of time) the movement of fragment in the field of contact may be interpreted as rolling on the contact surface, formed from the points, the radius is not constant. In a Fig. 15a and further in a Fig. 15 fat dotted line designates projections of virtual wheels to coordinate planes.
2. Passage of a package from several impulses is equivalent to rolling on the surface of contact final paleton the smooth wheels, generated from points of a piece, (a package soliton's) (see a Fig. 15b). After soliton's package has passed the piece again it becomes motionless. Such start-stop movement can be considered as possible, but it is slow. If soliton's packages follow with a final interval the final curve pieces remain motionless during the final time.
3. If impulses (soliton's) follow continuously one by one only contact points are motionless instantly. All other points move. Thus, moving along a piece of the continuous (running) wave is equivalent to rolling on the surface of contact which is constantly formed from the points of the piece of wheel infinite paleton, moving along the basic plane (see a Fig. 15c)

Lead a reasoning has allowed to construct a kinematic model of rectilinear moving modes of the snake or the snakelike robot. The form and amplitude of an impulse are essential to a quantitative estimation of the moving object speed; maintenance of motionless contact suffices for basic realization of movement on the final number of pieces moving along a material piece from "tail" to "head". Clearly, that movement of the real device in such mode is possible only at performance of static stability conditions under influence of external forces. These conditions can be executed due to finiteness size of the contact area. Besides similar sort of movement can be widespread for a curve laying on any surface. Moving speed of the cross-section wave laying on a cylindrical surface, will be still defined by moving speed of a projection of a "tail" point on a basic surface (see a Fig. 15d). The image of sliding wheels not only reflects an essence movement, but also allows to lead generalizations and to construct

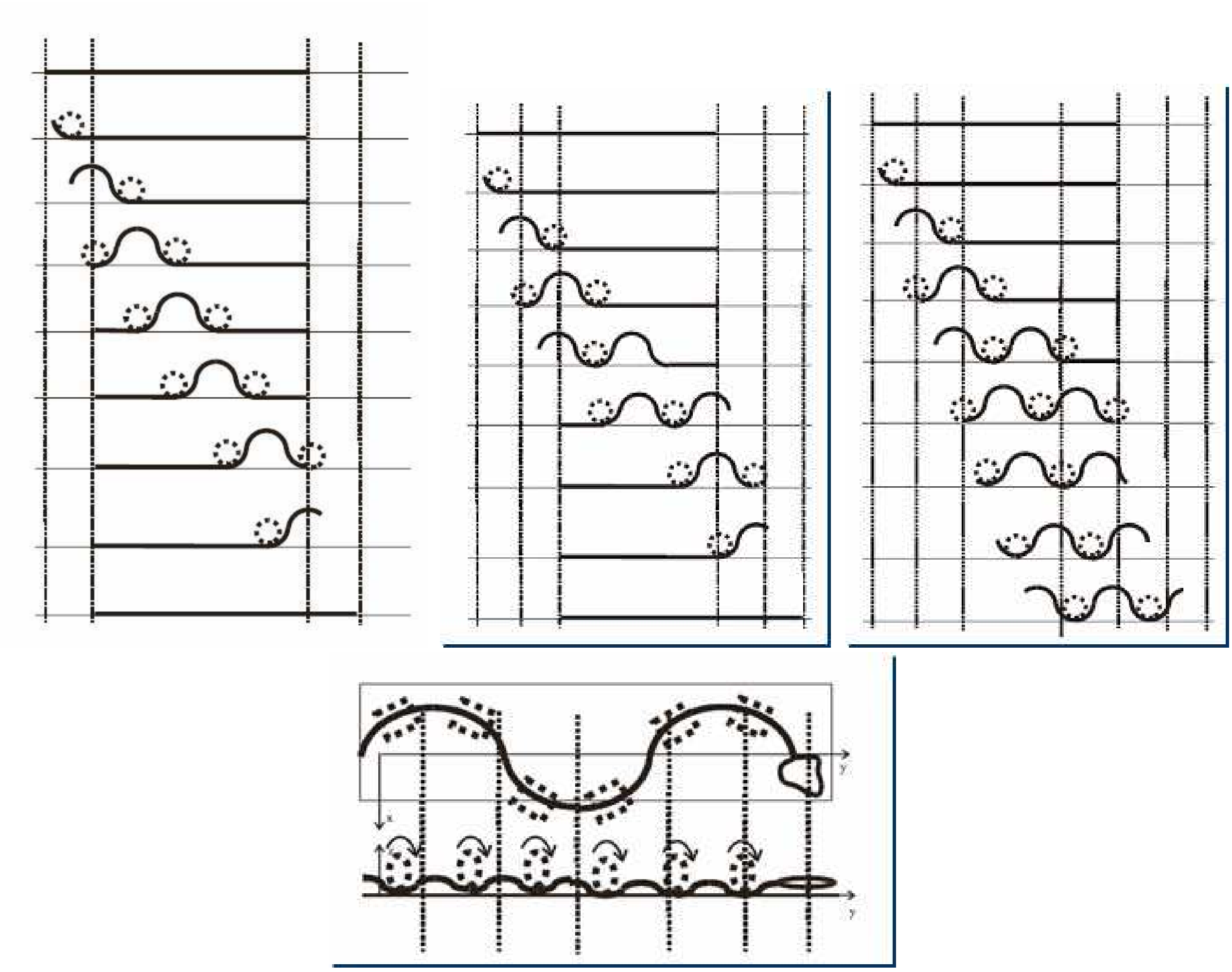


Figure 15: a,b,c,d

models for others locomotion modes.

Moving without creeping a deformable. Not extensible finite piece of the continuous - differentiated curve of the variable form, laying on the curvilinear cylindrical surface of the perpendicular plane, containing the generatrix, is the problem that arises in connection with instability of balance and movement of the "heavy" curve in the plane of gravity, thus, deplanation of the latter force is necessary. The form curve deplanation (projections of a spatial curve to a basic plane) and a minimum quantity of contact points is defined by static and dynamic balance conditions. In particular for static stability performance it is enough to provide contact in three points of a basic plane, which are not laying on one straight line. For preservation of spatial orientation of "wheels" it is necessary to move contact points along the material curve. This requirement is satisfied with the task of the second running wave of twice greater frequency and orthogonal to the first running wave.

The task of the small vertical amplitude wave and the big amplitude of lateral bending leads to a model locomotion modes - lateral bending bending on the rough surface.

The analysis of movement schemes allows offering one more scheme to arrange "wheels" which is formed at addition of two perpendicular running in one direction "from head to tail" phase shifted waves (a Fig. 15). Such scheme of movement at shift of phases differing from a half of the period leads to the lateral undulation model that is the most energetically favourable, mode of movement. All modes which are occur in nature can be considered within the limits of the offered scheme ”running waves - sliding wheels”

5 Mathematical model of snakelike robots

Snakelike robots locomotion control starts from a choice of the method to form the wave. The formula which described serpenoid curve is used. The essence of all mathematical evaluation is reduced to selection of parameters for the traveling wave formula.

$$\Phi(t) = A \cdot \sin\left(\varpi \cdot t + \varphi\right) \tag{3}$$

or

$$\Phi(t) = A \cdot \cos\left(\varpi \cdot t + \varphi\right) \tag{4}$$

$$where: A, \varpi, \varphi - parameters$$

At such approach to the control a choice of parameters are carried out in accordance with the mechanical model. But as the majority of researches have specified this problem as badly stated, that is why they have solved to use various algorithms for navigation of optimum parameters. Most often meeting methods: Random Search, Hill climbing, Simulated, Annealing, Neural Nets, Response Surface Methods, Genetic Algorithms, Trigonometric forms, Fourier, Parametric curves, Bayesian optimization algorithms, reinforcement learning in evolutionary computations.

Method of parameter selection with the use of genetic algorithms got the greatest propagation. The greatest propagation was gained with a trial and error method of parameters with usage of genetic algorithms. These algorithms are used by Kewin Dowling, CONRO research group,Ivan Tanev, Biorobotics Lab of Carnegie Melon University, Mark Yim from PARK. And others.

5.1 Genetic algorithms

GP is a domain-independent problem-solving approach in which a population of computer programs (individuals’ genotypes) is evolved to solve problems. The simulated evolution in GP is based on the Darwinian principle of reproduction and survival of the fittest. The fitness of each individual is based on the quality with which the phenotype of the simulated individual is performing in a given environment. The major attributes of GP - function set, terminal set, fitness evaluation, genetic representation, and genetic operations are elaborated in the remaining of this Section [10].

Function Set and Terminal Set. In applying GP to evolution of Snakebot, the genotype is associated with two algebraic expressions, which represent the temporal patterns of desired turning angles of both the horizontal and vertical actuators of each morphological segment. Since locomotion gaits are periodical, we include the trigonometric functions sin and cos in the GP function set in addition to the basic algebraic functions. The choice of these trigonometric functions reflects our intention to verify the hypothesis (first expressed by Petr Miturich in 1920’s) that undulative motion mechanisms could yield efficient gaits of snake-like artifacts operating in air, land, or water. Terminal symbols include the variables time, index of morphological segment of Snakebot, and two constants: Pi, and random constant within the range. The main parameters of the GP are summarised in Table

Category	Value
Function set	sin, cos, +, -, *, /
Terminal set	time
Population size	200 individuals
Selection	Binary tournament, ratio 0.1
Elitism	Best 4 individuals
Mutation	Random subtree mutation, ratio 0.01
Fitness	Velocity of simulated Snakebot during the trial
Trial interval	180 time steps, each time step account for 50ms of "real" time
Termination criterion	(Fitness > 100) or (Generations > 30) or (no improvement of fitness for 16 generations)

table 1

The rationale of employing automatically defined function (ADF) is based on empirical observation that the evolvability of straightforward, independent encoding of desired turning angles of both horizontal and vertical actuators is poor, although it allows GP to adequately explore the search space and ultimately, to discover the areas which correspond to fast locomotion gaits in solution space. We discovered that (i) the motion patterns of horizontal and vertical actuators of each segment in fast locomotion gaits are highly correlated (e.g. by frequency, direction, etc.) and that (ii) discovering and preserving such correlation by GP is associated with enormous computational effort. ADF, as a way of introducing modularity and reuse of code in GP is employed in our approach to allow GP to explicitly evolve the correlation between motion patterns of horizontal and vertical actuators as shared fragments in algebraic expressions of desired turning angles of actuators. Moreover, the best result was obtained by (i) allowing the use of ADF as a terminal symbol in algebraic expression of desired turning angle of vertical actuator only, and (ii) by evaluating the value of ADF by equalizing it to the value of currently evaluated algebraic expression of desired turning angle of horizontal actuator.

Fitness Evaluation. The fitness function is based on the velocity of Snakebot, estimated from the distance which the center of the mass of Snakebot travels during the trial. The real values of the raw fitness, which are usually within the range (0, 2) are multiplied by a normalizing coefficient in order to deal with integer fitness values within the range (0, 200). A normalized fitness of 100 (one of the termination criteria shown in Table 1) is equivalent to a velocity which displaced Snakebot a distance equal to twice its length.

Genetic Operations. Binary tournament selection is employed - a robust, commonly used selection mechanism, which has proved to be efficient and simple to code. Crossover operation is defined in a strongly typed way in that only the DOMnodes (and corresponding DOM-subtrees) of the same data type (i.e. labeled with the same tag) from parents can be swapped. The sub-tree mutation is allowed in strongly typed way in that a random node in genetic program is replaced by syntactically correct sub-tree. The mutation routine refers to the data type of currently altered node and applies randomly chosen rule from the set of applicable rewriting rules as defined in the context-free grammar of strongly typed GP.

ODE. We have chosen Open Dynamics Engine (ODE) to provide a realistic simulation of physics in applying forces to phenotypic segments of Snakebot, for simulation of Snakebot locomotion. ODE is a free, industrial quality software library for simulating articulated rigid body dynamics. It is fast, flexible and robust, and it has built-in collision detection. Figure - 1 Fitness convergence characteristics of 10 independent runs of GP for cases where

fitness is measured as velocity in any direction (a) and snapshots of sample evolved best-ofrun sidewinding locomotion gaits of simulated Snakebot (b, c), viewed from above. The dark trailing circles depict the trajectory of the center of the mass of Snakebot. Timestamp interval between each of these circles is fixed and it is the same (10 time steps) for both snapshots. Figure - 2 Trajectory of the central segment (cs) around the center of mass (cm) of Snakebot

for a sample evolved best-of-run sidewinding locomotion (a) and traces of ground contacts (b).

5.2 The determined approach

There are some other method of mode movement forming offered by Ivanov A.A. On the base of the approach mentioned above a kinematics model has been realized. All basic movement modes of the snakes could be formalized by representation of the equation defining the shape of its body at any moment. To describe an undulating motion of the snakelike robot it was offered to use sinusoidal dependence of an angle from the are coordinate. Unlike all previous works, the author specially introduced two perpendicular waves tat allow to organize relocation of the contact points with the motionless plane length wise the axis of the snakelike robot, by implementing a mode of instantaneous rolling of the contacting section curve (an equidistant surface) along the latter plane. The main movement "Takes place in the contact plane", vertical movement is only responsible for "support areas" organization; these areas allow to avoid sliding, even the construction has no wheels sliding is the most unfavorable mode of movement. Thus, limitations on relocations in the contact points completely correspond with the classical case of nonholomonic constraint at right body rolling on the rough surface [4].

The uniform kinematic description of all modes of locomotions can be lead with use of concept of virtual wheels formed at wave movement of a flexible body. The part of modes is realized at a mode of a rolling without sliding, a part at a rolling with creep-ing lengthways "rims of a wheel " (a nonholonomic mode of a rolling). In a Fig. 16 schemes observable in the nature limblesses gaits and the virtual wheels formed for their realizations are presented.

It is necessary to note that the nonholonomic mode (e) is realized by the majority of kinds of snakes at movement on slippery surfaces with use of lateral support (heterogeneousnesses) or created due to bending in a vertical plane of the isolated zones of a rolling with creeping along a body and an immovability in a perpendicular direction. Speed of moving of snakes is proportional to speed of distribution of a running spatial wave of bending. This wave forms of two flat waves which perpendicularity is defined by features of a structure of a backbone of the snake 4. The direction of movement of the snake depends on a combination of a direction of distribution along a body of a running wave, lengths and relative shift of phases of combined waves. Use of representation about virtual wheels, formed in zones of contact of a flexible body with a bearing surface, allows to define unequivocally admissible combinations of parameters of a wave and a possible direction of its movement. Curvature of a body in the field of con-tact defines radius of a virtual wheel which can be adapted for deformation properties of a bearing surface. Speed of moving of the center of masses of periodically bent body leaning a rough plane, is defined by the formula 5

$$\bar{v}_c(t) = -\dot{x}[\bar{\rho}(L_f, x(t))/L_f - \bar{\tau}(s_0)] \tag{5}$$

in which it is designated L_f - length of a flexible body multiple to length of a wave of a wave of bending running along a body, $\bar{\rho}(L_f, x(t))$ - radius-vector of a final point of a curve concerning its beginning, $\bar{\tau}(s_0)$ - tangential ort in a point of a contact of a curve and basic plane, x and $\dot{x}$ - are coordinate and speed of front of a running wave, $\bar{v}_c(t)$ - speed of the center of masses. At realization a nonholonomic mode of a rolling with creeping the moving of the center of masses occurs in a direction of a total vector tangential orts in two next points of contact and its speed is defined by the formula

$$v_c(t) = \dot{x}[|\bar{\rho}(L_f, x(t))|/L_f - 1] \tag{6}$$

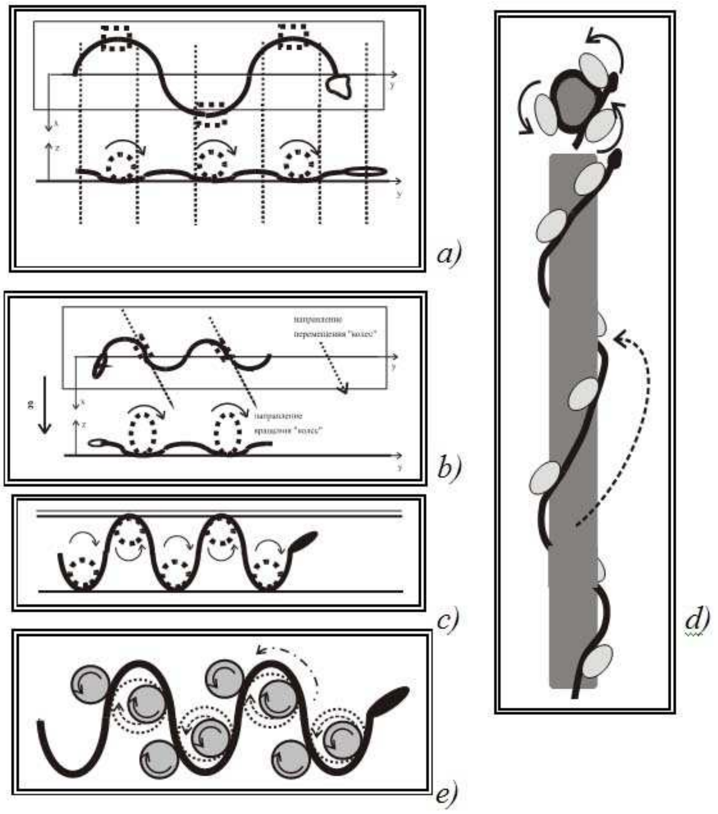


Figure 16: Schemes of realization of various modes of locomotions of snakes. Rectilinear movement by bending without sliding (a), lateral movement sidewinding (b), concertina in channel (c), movement on a core (d), propulsion (with creeping) (e)

Limblesses universality in relation to environment of movement, static stability of the majority of sold modes of movement, high adaptibility at relative simplicity of an anatomic structure became the reason of heightened interest of researchers and the prosecute subjects of development of independent mobile robots engineers to a phenomenon of movement without use of limbs.

Now in the world there are some research groups developing the SNAKELIKE ROBOT with wheelness modules . Characteristic feature of a design of these robots is the module from the rigid cases connected by the revolute hinge of rotation with the servo-driver. The snakelike structure is formed of consistently connected modules with alter-nation of directions of axes of rotation. It allows to set the form of bending of a circuit of modules in the form of a broken line in each of orthogonal planes. For detemination of the form of a flat running wave sine wave dependence for $\varphi_{l\,i}$ interlinks angles of turn around of l_i axes of hinges of identical orientation is used

$$\varphi_{l\,i} = 2\varphi_l^* \sin(\pi\eta/N_w)\cos(2\pi\eta(i - \nu N_w t)/N_w + \delta_l) \tag{7}$$

where φ_l^* is amplitude of an absolute corner of turn of the module around of a normal to wave plane axis, N_w is the length of a horizontal wave expressed in links, ν is frequency of bending of the SNAKELIKE ROBOT body, η is the ratio of length of a horizontal wave to length of a wave in a considered plane, δ_l is shift of phases of orthogonal waves. The formula of a kind (3) for the first time has been used Hirose for the task of the form of the plane wheel robot. As it has been shown above.

6 Hardware realization control

Realization hardware level of control as a rule represents two subsystems: the first subsystem (the high level control), as a rule is a personal computer or the similar device, the second subsystem (the low level control) stands onboard of the snakelike robot and represents the microcontroller or a little bit which are responsible for importation of data from the high level and sensors, data processing and control mover which are used with the snakelike robot[4].

The variant when the low level control itself control all robot and plans movement relying only for sensors, does not exist yet. In connection with that computing capacities are necessary for this purpose very big, and such microcontrollers, in a small dimension with small power consumption simply do not exist.

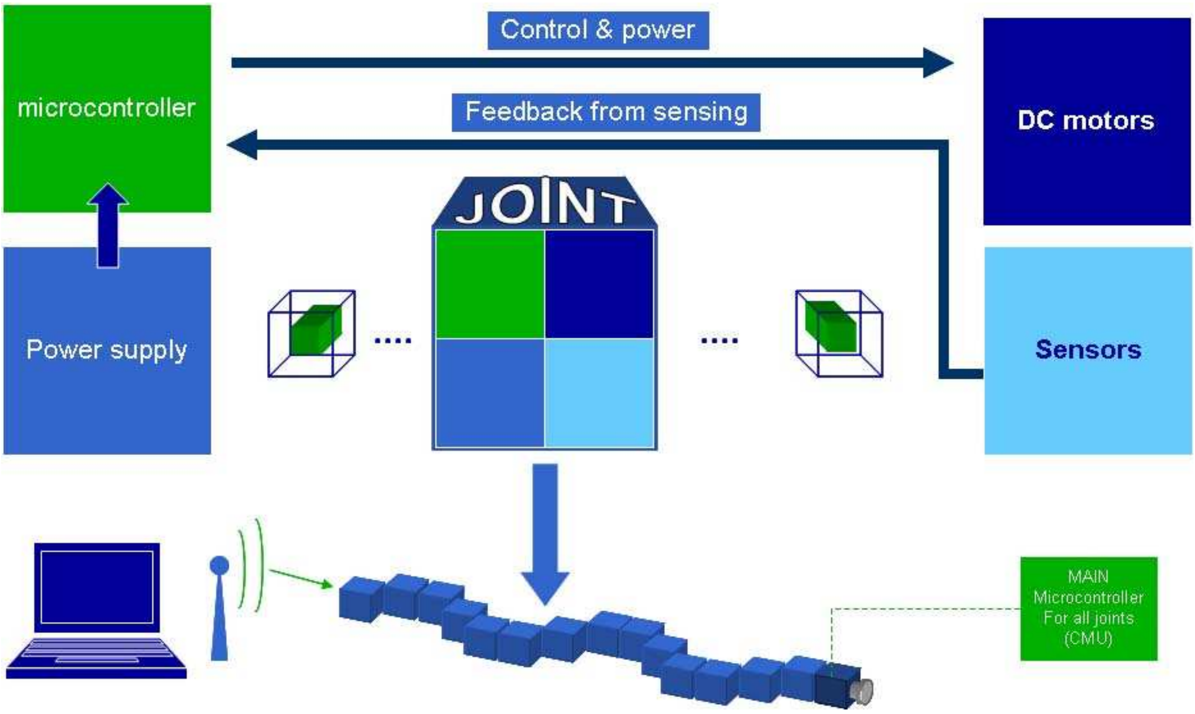


Figure 17: Hardware realization control

For this reason the high level control which is necessary control the low level control and high-grade data processing from sensors. On the high level locomotion modes are forming and sending to low level. The locomotion planning is a very hard work and these problem not solve properly yet.

For connection subsystems ar rule use some simple protocol connection: hard-wire and wireless. Hard-wire channels: RS-232 and CAN use SankeWheel-1, Dowling, Michalachi, CMU and some others Wireless: WiFi, ZigBee, Bluetooth use BIRG, CMU, Miller and others.

7 Snakelike robot CRDI RTC

Snakelike robot CRDI RTC was made in 2004 year by command of professor Saint Petersburg State Polytechnical University Ivanov A.A. For demonstration of dynamic model of the coiled robot which has been offered in Ivanov A.A. works in 2002. Ivanov used the biomorphic approach which is one of the cores in a robotics. Ivanov offered kinematic model of movement of a snakelike body gives a rationale for creation of the snakelike robot design as much as possible approached to biological analogue. Moving to space of chain structure of the unified modules

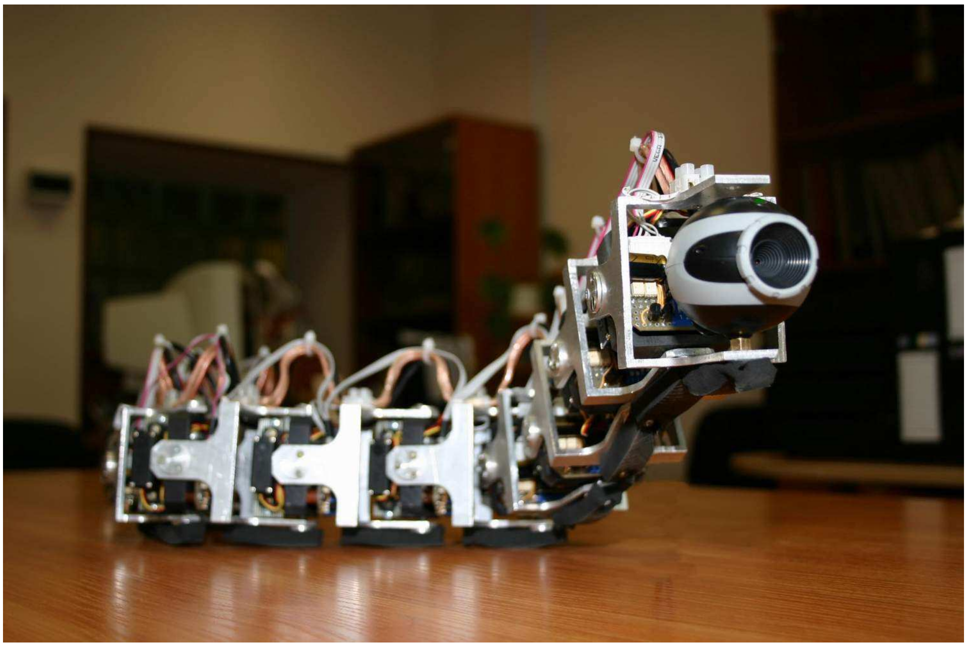


Figure 18: SnakeWheel-1

can be organized due to creation by internal drives of waves of bending running along a circuit with a length less than its length. For maintenance of smoothness of movement it is necessary to provide a mode of rolling the rigid module on a basic surface and whenever possible smooth transition of a spot of a support from one module on another. The combination of the running waves located in two perpendicular planes in one direction allows to carry out moving on any surfaces without use of a traditional wheel mover forming from a circuit of modules "flexible wheels " desirable curvature in necessary number of basic zones and creating on analogies to a caterpillar wheel a mover "a snakewheel ". Such mover allows to move snakelike structure even on contact surfaces with small friction when basic zones slip in tangential to a direction of a vector of speed of movement of a wave a direction. The structure of the vertebral joint of the snake provides the limited relative turn adjacent vertebras around of two orthogonal axes not supposing twisting of a vertebras. The range of change of a corner of relative turn vertebras around of a vertical axis is much more than range of change of a corner of turn around of a horizontal axis. Technical realization of such joint can be executed in the form of the universal orthogonal hinge with different ranges of mobility. The offered ideology of formation of movement the snakelike robot gives a basis for an estimation of power characteristics of drives of active hinges. At realization of various modes of snakelike movements drives of the multijoint SNAKELIKE ROBOT should provide the moments necessary for maintenance demanded for movement in the chosen mode of the form of an axial line. From a Fig. 1 it is visible that at various modes of movement on a plane the maximal moment M_l created by force of weight concerning an axis l a perpendicular axial line arises in a point of a support at deduction free tail (or head) parts before the beginning of formation of a new zone of contact. For a smooth piece curve this moment is defined by the formula

$$M_l = \mu_{form} m_w g L_w / 2 \tag{8}$$

where m_w is the mass of a kept console part, L_w is length of this part, $mu_{form} < 1$ is the factor depending on the form of the kept curve. For a circuit of the rigid parts connected by hinges the formula 8 allows to write down an estimation from below for the moment of a drive of a horizontal axis of the universal hinge

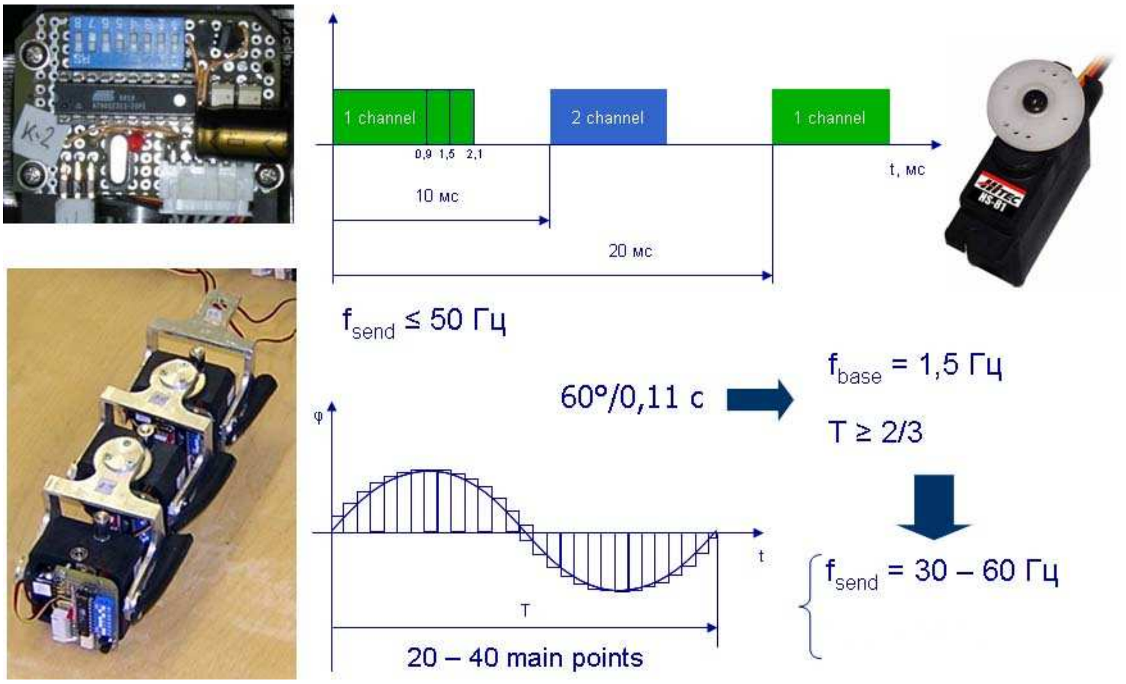


Figure 19: Hardware realization control

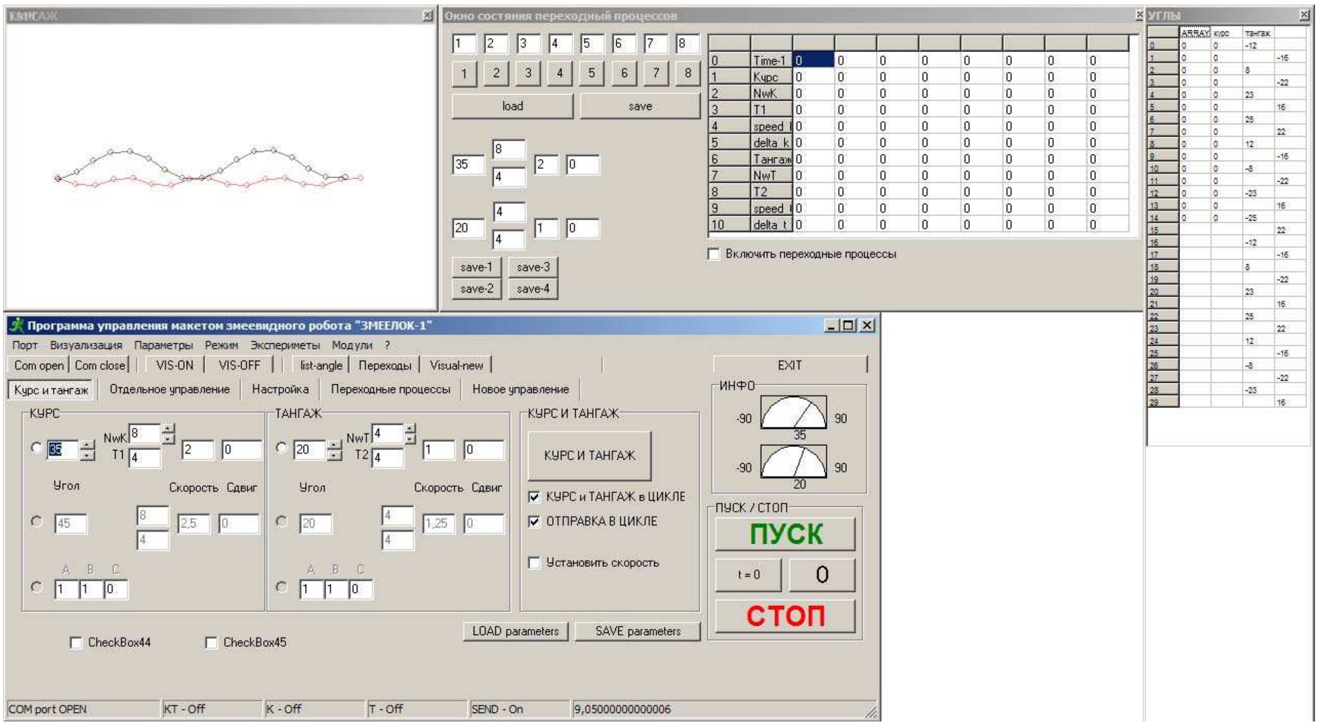
$$\mu_{form}\Delta\, N^2(2 + P_u/P_{ac})/2 \leq K_p M_{ac}/P_{ac} \tag{9}$$

Here Δ is center to center universal hinges distance, N is quantity of modules kept on weight, P_u/P_{ac} is the attitude of weight of the module without taking into account weight of two servo-drivers of the hinge to weight of the servo-driver, M_{ac}/P_{ac} is the ratio of the moment developed by a drive to its weight, K_p is factor of transfer of an additional reducer.

The high-speed parameters of a drive necessary for performance of movement set by the formula 7 will be defined from an inequality for the maximal angular speed of working off of a drive $omega_{servo}$ and the maximal angular speed of rotation in the hinge necessary for realization of bending movement of demanded frequency and amplitude

$$4\varphi_l^* \sin(\pi\eta/N_w)\pi\eta\nu \leq \omega_{servo} \tag{10}$$

In view of the biomorphic approach and restrictions 9 and 10 the universal hinge of the module the snakelike robot also has been developed. As servo-drivers of both axes of the hinge serial steering machines HS-81MG by mass 19g with the maximal moment 26Ncm at a voltage 4.8v and speed of working off $60^\circ/0.11$s have been used. For maintenance of performance of restriction (5) on a horizontal axis the additional reducer with $K_p = 4$ has been established. Reduction of a range of change of an angle of turn in the horizontal hinge does not contradict to the biomorphic approach. On basic cases-plugs the rubber support providing coupling with the bearing surface in a phase reference of their movement and smoothness of moving of a contact spot along a body the snakelike robot are established. In MSC.ADAMS software parametrical research of dynamics of the virtual prototype 16 links snakelike robot collected of modules with two DOF universal hinges has been executed. The basic modes of snakelike locomotions on a plane have been realized and correctness of the made estimations and the chosen technical decisions is confirmed. Under electronic drawings of the virtual prototype the SNAKELIKE ROBOT breadboard model has been made. Overall dimensions of the module 63õ63mm at distance centre to centre hinges 70mm. Mass of one module 200g.



Программа управления макетом змеевидного робота "ЗМЕЕЛОК-1"

0	Time-1	0	0	0	0	0	0	0	0
1	Курс	0	0	0	0	0	0	0	0
2	NwK	0	0	0	0	0	0	0	0
3	T1	0	0	0	0	0	0	0	0
4	speed	0	0	0	0	0	0	0	0
5	delta_k	0	0	0	0	0	0	0	0
6	Тангаж	0	0	0	0	0	0	0	0
7	NwT	0	0	0	0	0	0	0	0
8	T2	0	0	0	0	0	0	0	0
9	speed	0	0	0	0	0	0	0	0
10	delta_t	0	0	0	0	0	0	0	0

☐ Включить переходные процессы

	ARRAY	курс	тангаж	
0	0	0	-12	
1	0	0		-16
2	0	0	8	
3	0	0		-22
4	0	0	23	
5	0	0		16
6	0	0	25	
7	0	0		22
8	0	0	12	
9	0	0		-16
10	0	0	-8	
11	0	0		-22
12	0	0	-23	
13	0	0		16
14	0	0	-25	
15				22
16			-12	
17				-16
18			8	
19				-22
20			23	
21				16
22			25	
23				22
24			12	
25				-16
26			-8	
27				-22
28			-23	
29				16

☑ КУРС и ТАНГАЖ в ЦИКЛЕ
☑ ОТПРАВКА В ЦИКЛЕ
☐ Установить скорость
☐ CheckBox44
☐ CheckBox45

COM port OPEN | KT - Off | K - Off | T - Off | SEND - On | 9,05000000000006

Figure 20: SnakeCharmer: program for control SnakeWheel-1

For formation PWM of signals of control by work of pair servo-drivers onboard each module the control card on the basis of microcontroller AT90S2313 (Atmel) is placed. Individual number of the module is established by the switch placed to the same card ac-cording to a place of the module in a consecutive circuit. In a Fig. 2 the photo of assembly from three modules with control card onboard is presented. All modules are connected in parallel to matching network MAX232. A power feed of drives is realized on two wire trunk from a stationary source. The connected through COM-port to a personal computer SNAKELIKE ROBOT digital control are carried out the program which calculates values of ancles of turn for a present situation of time in all 30 hinges of rotation under formulas 7 and forms and sends on COM port a package of 63 bytes of commands with a speed 115.2 Kbit/sec. The package acts simultaneously on inputs of all of 15 microcontrollers and from it the set addressed to corresponding module gets out of four bytes defining the duration of the generated by microcontroller operating impulses. For the chosen speed of data transmission the packages sending off interval is defined only by frequency of the SNAKELIKE ROBOT bending and accuracy of approximation of a periodic signal of this frequency as for the chosen servo-driver of restriction ??10) suppose frequencies no more 1.5Hz, and the number of steps (settlement points) for the period of fluctuations gets out no more than 20.

Tests of a breadboard model during which theoretically predicted modes of moving on a plane are realized all have been lead. In a Fig. 20 the photo of an operating breadboard model the SNAKELIKE ROBOT in one phases of movement is presented by a lateral undulatin. Speeds of moving are correlated with generated frequency of bending and reach 4.4cm/sec at movement by a sidewinding mode with frequency of bending 0.5Hz/. Essential influence on speed of moving is rendered with quality of a supporting surface and a material and the form of supports of modules. Speed of drives at movement under loading decreases approximately twice. Power consumption of a breadboard model reaches 40W. Conclusion. The modern devices of the wireless communication allow to realize two-side transmission of the information and control signals. Problem of the portability of device is common for the autonomous mobile

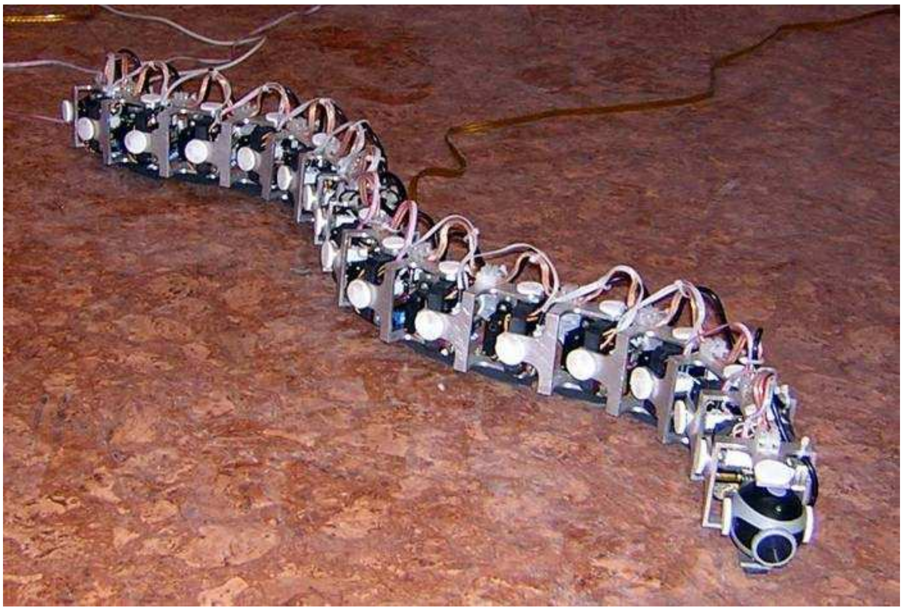


Figure 21: Snakewheel-1: lateral undulation

robots and can be solved with the ad-vent of the light accumulators of large capacity. The problem of a device motion control on a complicate condition can be solved within the bounds of the ”creation isolated contact zones” paradigm and its coordinate motion organization. The advantages of the construction hyper redundancy give base to speak about a possibility of use snakelike robot as an informational adaptive mobile platform.

8 Conclusions

In this report we’ve discussed how to control snakelike robots, about advantages and disadvantages. Some descriptions with examples of snakelike robots were introduced. We discussed mechanic and mathematical model of snakelike robots these models may be very helpfully for locomotion control of snakelike robots and for understanding how to do such control better. And at the end we speak about first snakelike robot in Russia witch made in CRDI RTC in Student Engendering Design Office under the direction of professor Ivanov A.A. On these example we examine how to control it, which problems researchers solved when design such robots.

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