Building Abstraction Hierarchies from Unbounded-arity Relations

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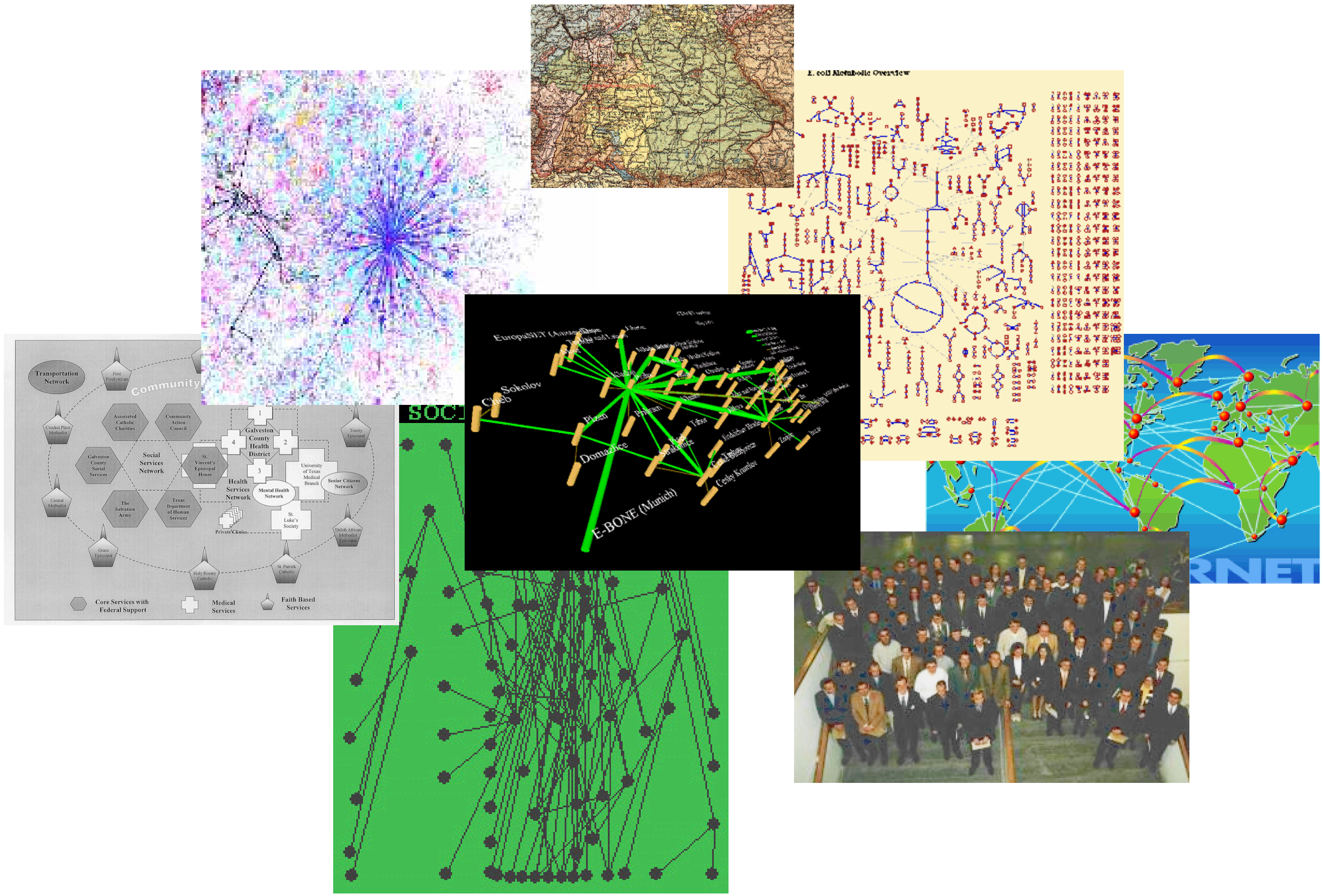
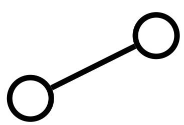
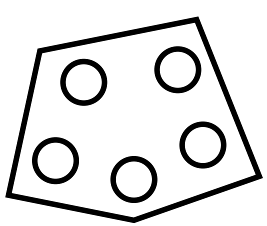
Real World Networks



Basic Concepts of Dealing with Real World Networks

Modeling the World

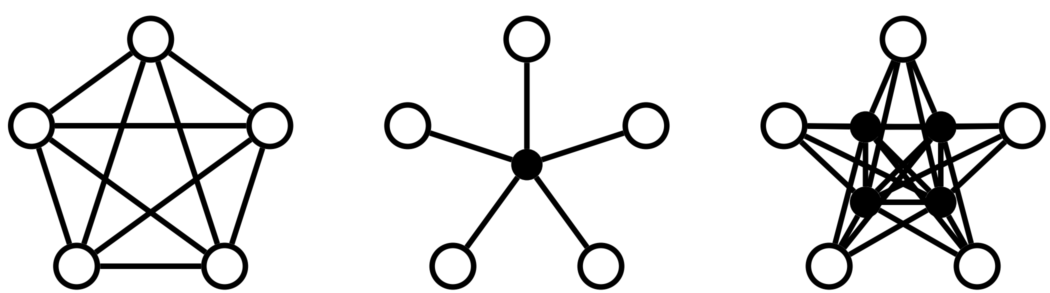
real world networks
=
entities and relations

two-arity 	symmetric: • who-knows-who asymmetric: • hyperlink graph • institutional hierarchy
unbounded-arity 	• Hollywood graph • social groups • co-authorship (LEABIB)

Modeling Relations

relations		hyper graphs		simple graphs
entities subsets	$\Rightarrow$	vertices hyperedges	$\Rightarrow$	vertices edges

- Transformation of entities to vertices (multiple occurrence, spelling)
- Introduction of weights for the hyperedges (size of subset, elements of subset)
- Transformation of hyperedge to edge



How to Deal with Large Data

Input properties:

- large-scale (10^5 authors, 10^{10} web pages)
- fuzzy data (noisy, incomplete)

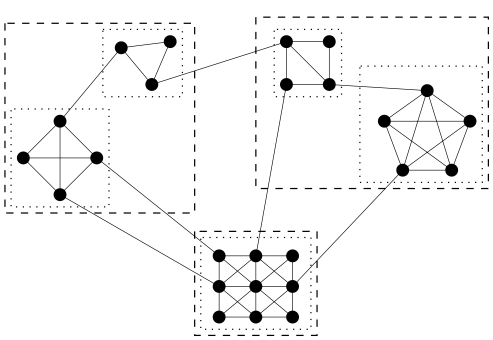
Opportunities:

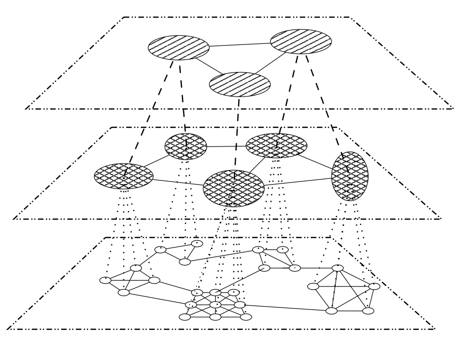
- reduction of the data

  $\rightarrow$ concentrating on a subset of entities

- abstraction of the data

  $\rightarrow$ representing groups of entities





Semantics of Hierarchies on Relations

Aim:

- groups of entities should represent strong relations between their elements
- relations between those groups should represent the relations between its group members

Applications:

- general multi-arity relations

  $\rightarrow$ social centers / structure

  $\rightarrow$ project management
- co-citation relation (LEABIB)

  $\rightarrow$ group detection (research, referees)

  $\rightarrow$ quality

  $\rightarrow$ query enhancement

How to detect groups?

- connectivity

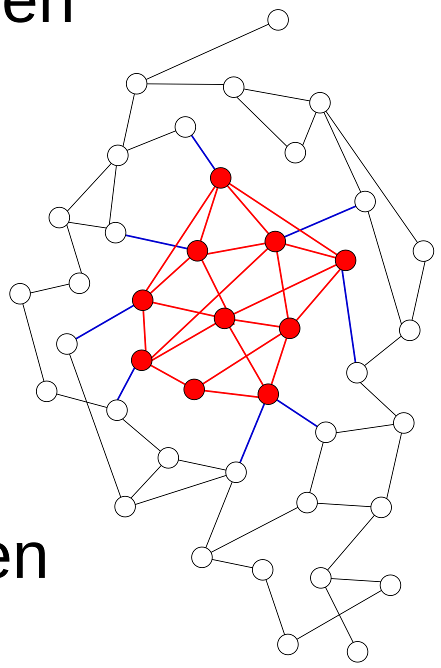
  $\rightarrow$ optimize number of disjoint paths between any two vertices in the group

- weighted Min-Cut (Ratio-Cut)

  $\rightarrow$ optimize the quantity: $\frac{\mathrm{Cut\_Size}(A,B)}{\|A\| \cdot \|B\|}$

- density based

  $\rightarrow$ optimize the number of relations between the entities in the group (e.g. $\|E\|$)



Density Based-Clustering(1)

Motivation:
- small-world characteristics (sparse graphs with dense substructures)
- looking for regions with significantly higher average degree

Problem:	γ-DENSE SUBGRAPH-PROBLEM (γ-DSP)
Input:	Graph G, $k \in \mathbb{N}$
Output:	Does there exist a subgraph G' of size k having at least $\gamma(k)$ edges

- $\gamma(k) = \binom{k}{2}$ γ-DSP $=$ CLIQUE $\in$ **NP-C**
- $\gamma(k) = 0$ γ-DSP $\in$ **P**

Complexity-Results

[H. et al. CIAC 2003]

Theorem. *Let* $\gamma : \mathbb{N} \to \mathbb{N}$ *be a function that is computable in polynomial time:*

1. *If* $\gamma(k) = k + O(1)$ *then* γ-DSP *is in* **P**.
2. *If* $\gamma(k) = k + \Omega(k^\epsilon)$, *for some rational number* $0 < \epsilon < 2$, *then* γ-DSP *is* **NP**-*complete.*

Theorem. *Let* $\gamma : \mathbb{N} \to \mathbb{N}$ *be a function that is computable in polynomial time.*

If $\gamma(k) \in k + \Theta(\log k)$ *and* γ-DSP *in* **NP**-*c, then*

NP$\subseteq$ **DTIME**$(n^{O(\log(n))})$.

An Abstraction Hierarchy for Unbounded-arity Relations

Our Approach

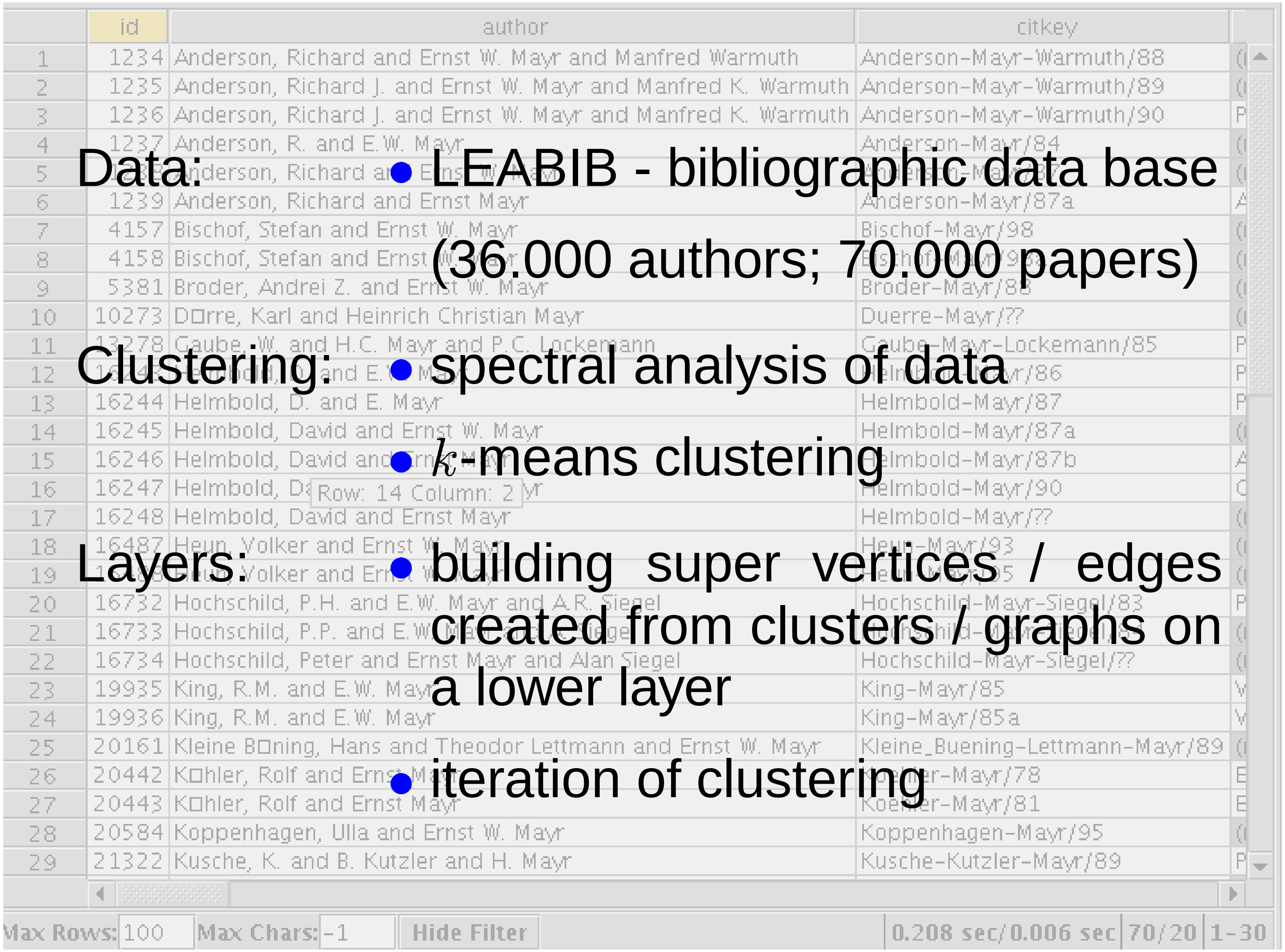
Data: • LEABIB - bibliographic data base (36.000 authors; 70.000 papers)

Clustering: • spectral analysis of data

• k-means clustering

Layers: • building super vertices / edges created from clusters / graphs on a lower layer

• iteration of clustering



	id	author	citkey
1	1234	Anderson, Richard and Ernst W. Mayr and Manfred Warmuth	Anderson-Mayr-Warmuth/88
2	1235	Anderson, Richard J. and Ernst W. Mayr and Manfred K. Warmuth	Anderson-Mayr-Warmuth/89
3	1236	Anderson, Richard J. and Ernst W. Mayr and Manfred K. Warmuth	Anderson-Mayr-Warmuth/90
4	1237	Anderson, R. and E.W. Mayr	Anderson-Mayr/84
5	[illegible]	[illegible]	[illegible]
6	1239	Anderson, Richard and Ernst Mayr	Anderson-Mayr/87a
7	4157	Bischof, Stefan and Ernst W. Mayr	Bischof-Mayr/98
8	4158	[illegible]	[illegible]
9	5381	Broder, Andrei Z. and Ernst W. Mayr	Broder-Mayr/88
10	10273	D□rre, Karl and Heinrich Christian Mayr	Duerre-Mayr/??
11	13278	Gaube, W. and H.C. Mayr and P.C. Lockemann	Gaube-Mayr-Lockemann/85
12	[illegible]	[illegible]	[illegible]
13	16244	Helmbold, D. and E. Mayr	Helmbold-Mayr/87
14	16245	Helmbold, David and Ernst W. Mayr	Helmbold-Mayr/87a
15	16246	[illegible]	Helmbold-Mayr/87b
16	16247	[illegible]	Helmbold-Mayr/90
17	16248	Helmbold, David and Ernst Mayr	Helmbold-Mayr/??
18	16487	Heun, Volker and Ernst W. Mayr	Heun-Mayr/93
19	[illegible]	[illegible]	[illegible]
20	16732	Hochschild, P.H. and E.W. Mayr and A.R. Siegel	Hochschild-Mayr-Siegel/83
21	16733	[illegible]	[illegible]
22	16734	Hochschild, Peter and Ernst Mayr and Alan Siegel	Hochschild-Mayr-Siegel/??
23	19935	King, R.M. and E.W. Mayr	King-Mayr/85
24	19936	King, R.M. and E.W. Mayr	King-Mayr/85a
25	20161	Kleine B□ning, Hans and Theodor Lettmann and Ernst W. Mayr	Kleine_Buening-Lettmann-Mayr/89
26	20442	[illegible]	[illegible]
27	20443	K□hler, Rolf and Ernst Mayr	Koehler-Mayr/81
28	20584	Koppenhagen, Ulla and Ernst W. Mayr	Koppenhagen-Mayr/95
29	21322	Kusche, K. and B. Kutzler and H. Mayr	Kusche-Kutzler-Mayr/89

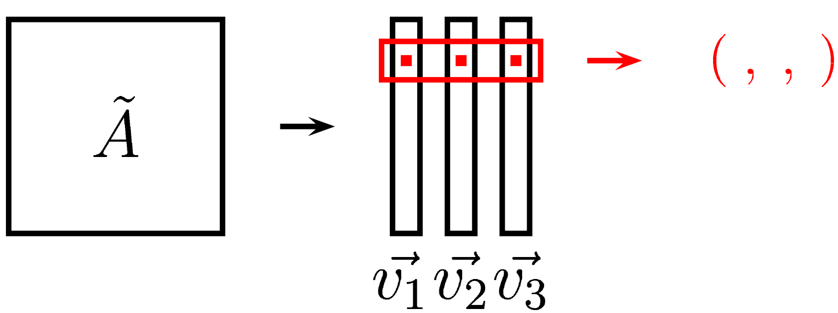
Clustering with Spectral Partitioning

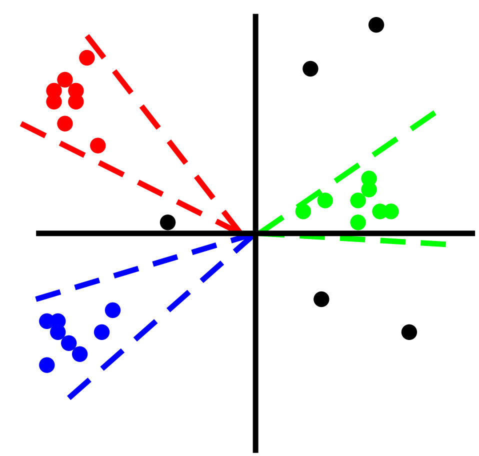
Idea:

- using SVD for low-rank approximation of (weighted) adjacency matrix
- transformation of n-dimensional data into a low-dimensional space

Advantage:

- fast approximation algorithms
- spectral partitioning and ratio-cut partitioning are correlated

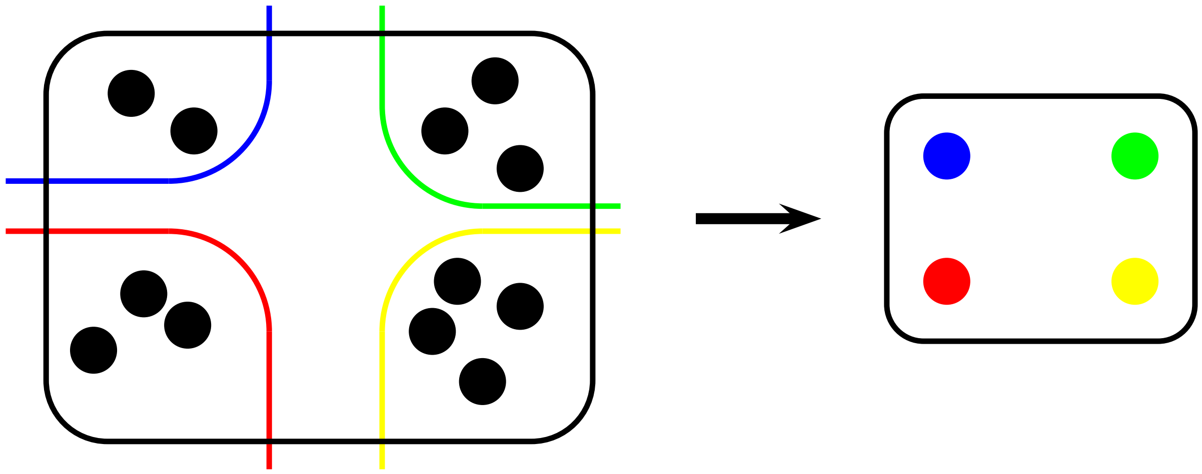




Building Levels

Input:	Hyper graph G, clustering C
Output:	Hyper graph G' at the next higher level

- vertices: clusters
- hyperedges: shrink old hyperedges



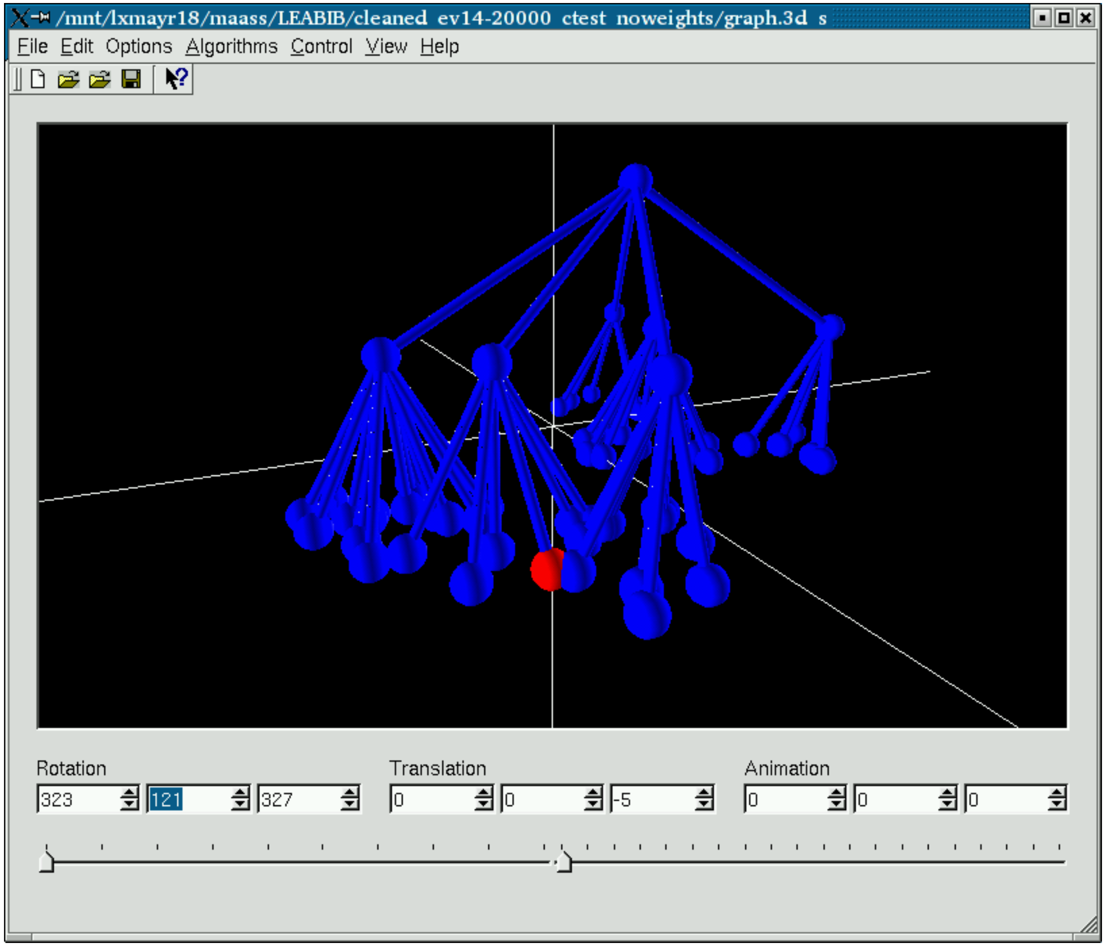
Analysis of Clusters

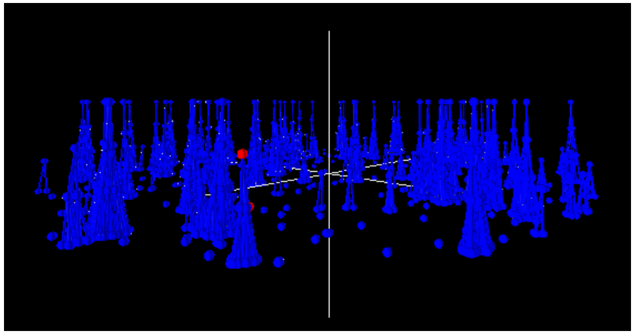
```
Cluster 191 on level 2:

Cluster   232 on level 1
Cluster   240 on level 1
Cluster   560 on level 1
Cluster   561 on level 1
Cluster   648 on level 1
Cluster   780 on level 1
```

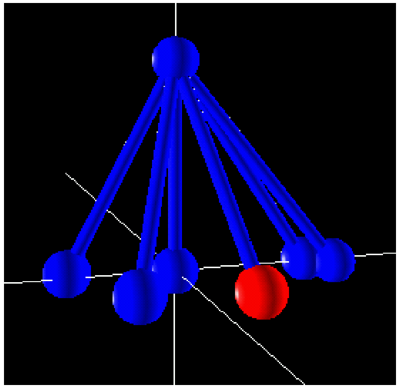
```
Cluster 232 on level 1:

 871  Mayr, E. W.
2564  Bischof, S.
6559  Heun, V.
7331  Lettmann, T.
7429  Koppenhagen, U.
```

```
Cluster 780 on level 1:

1110  Suen, S.
1216  Assaf, S.
1217  Upfal, E.
1304  Feige, U.
1348  Shamir, E.
1393  Broder, A. Z.
1394  Karlin, A. R.
3101  Frieze, A. M.
      ...
```

Visualization







Future Directions, What's Next(1)

Input Data

- difficulties: $\rightarrow$ noisy data
  $\rightarrow$ small data set
- tasks: $\rightarrow$ data cleaning
  $\rightarrow$ incorporation of further data

Modeling of hyper graphs

- representation of hyperedges
- usage of weights
- graph abstraction

Future Directions, What's Next(2)

$\boxed{\text{Dynamic hierarchies}}$

- modification of hierarchy
  - $\rightarrow$ union/split of clusters
  - $\rightarrow$ insertion/deletion of levels
- incorporation of further data
  - $\rightarrow$ insertion/deletion of vertices/edges
  - $\rightarrow$ modification of weight functions
- parametric clusters/hierarchies
  - $\rightarrow$ transformation/weights of hyperedges
  - $\rightarrow$ graph generation on each level