Affix Trees

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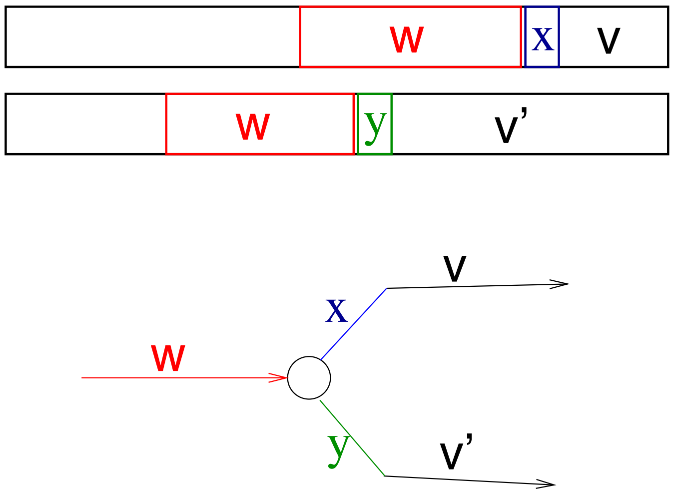
Agenda

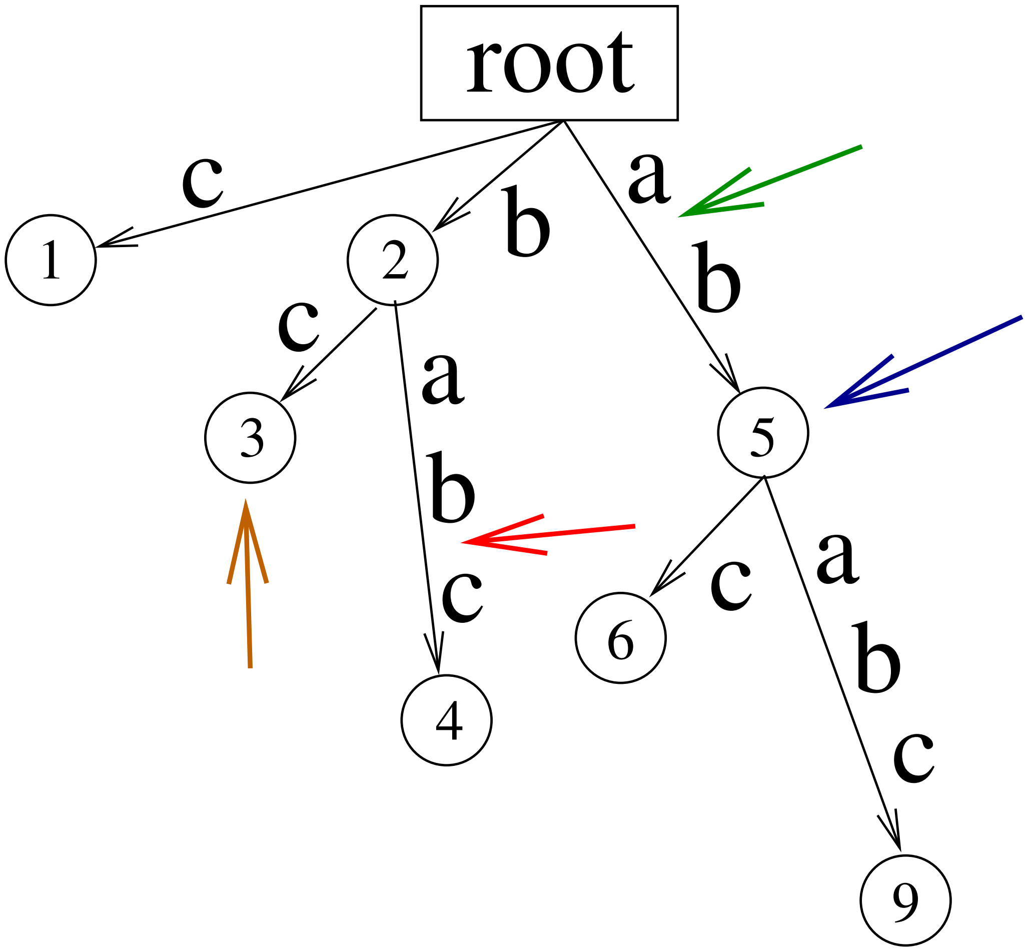
1. Introduction
2. Construction of Suffix Trees
3. Construction Affix Trees
4. Complexity

Important Suffix Tree Properties

- Representation of repeated substrings
- Right branching substrings are represented by branching nodes
- Each tree position represents a unique string
- Moving down in the tree extends the string, moving towards the root shortens the string.



Representation of Tree Positions with Reference Pairs



root + a

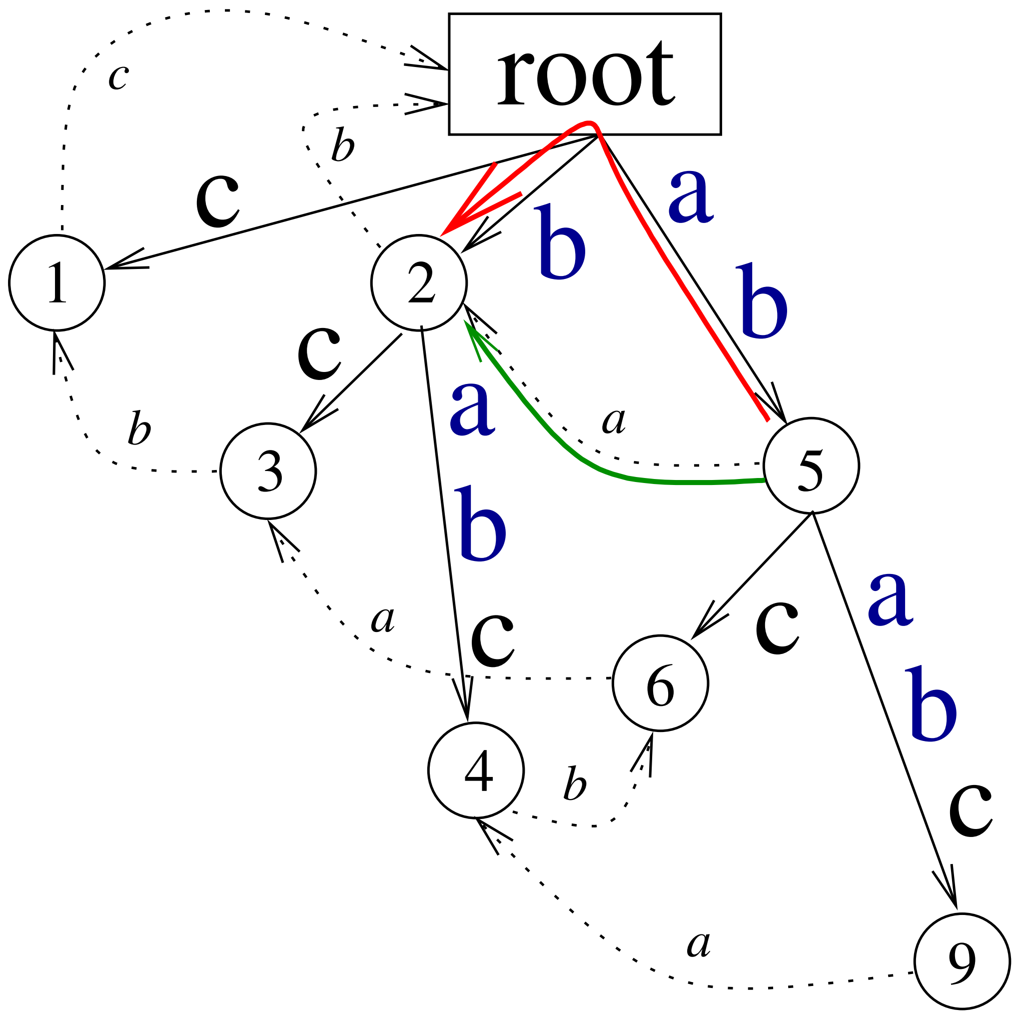
5 + ε

3 + ε

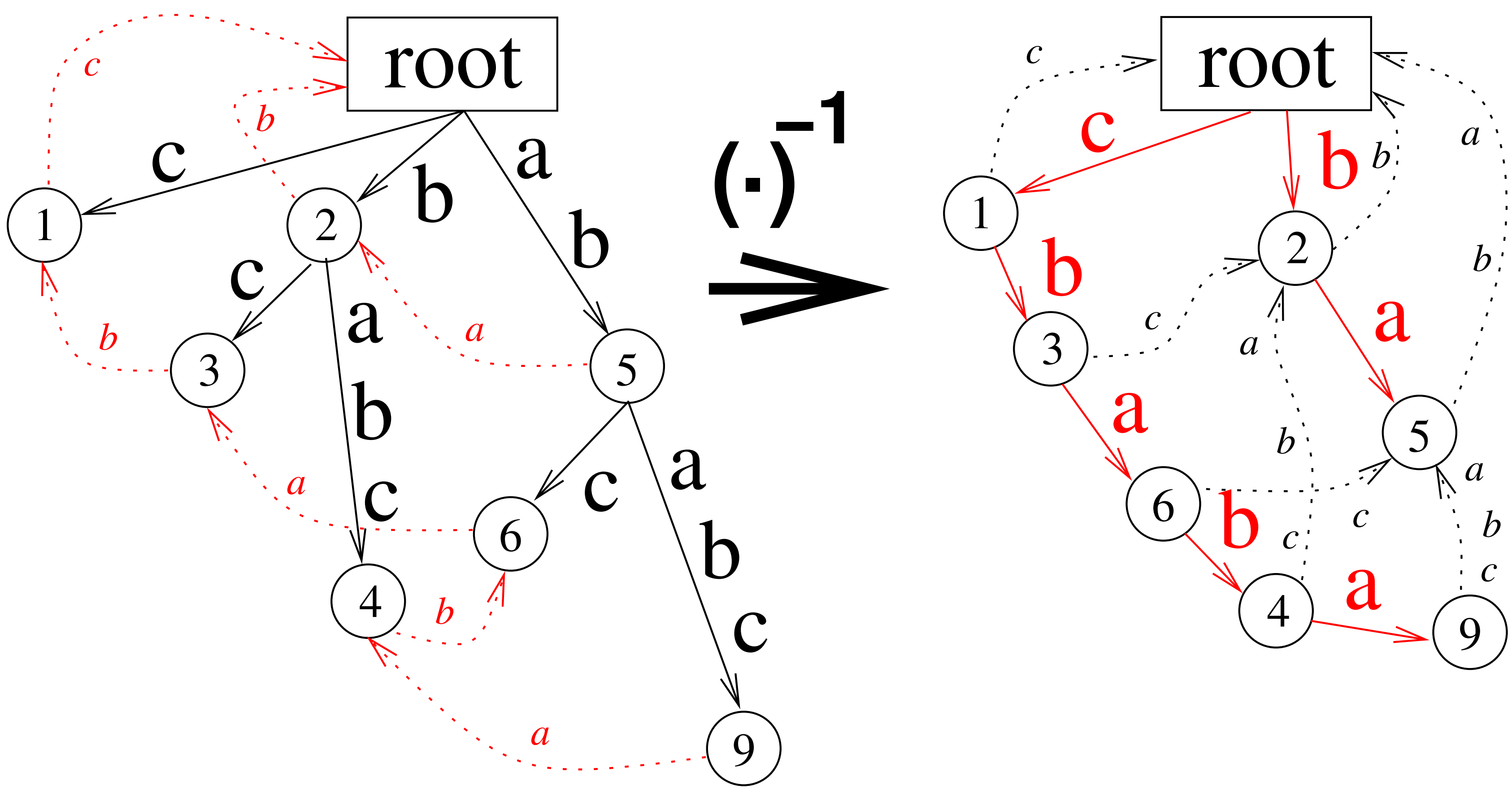
2 + ab

Suffix Links

- Two ways of "shortening" the represented substring abab at the front to come to the position of bab
- Suffix links operate at the front of the represented tree, while edges operate at the end

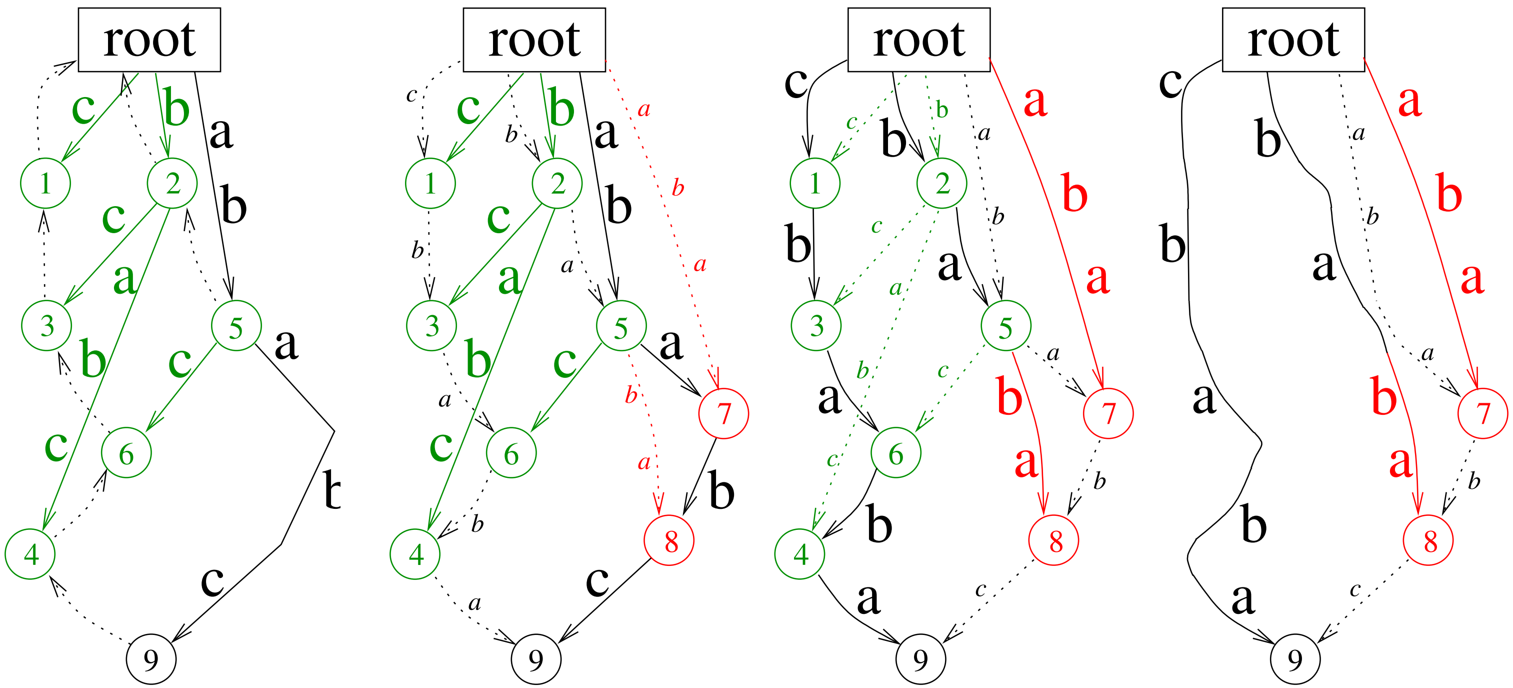


Reverse Tree

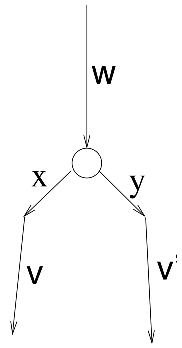


Affix Trees

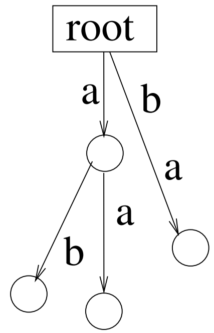
CST(ababc) CAT(ababc) CAT(cbaba) CST(cbaba)



Definition 1 (right branching and left branching). *A substring w of t is right branching (left-branching), if there w occurs at two different positions in t with distinct succeding (preceding) letters (w is r.b. in t, if $\exists x, y \in \Sigma, u, v, u', v' \in \Sigma^*. t = uw\mathrm{x}v \quad \wedge \quad t = u'w\mathrm{y}v' \quad \wedge \quad \mathrm{x} \neq \mathrm{y}$).*



Definition 2 (Σ^+-tree). *A Σ^+-tree T is a rooted, directed tree with edge labels from Σ^+. For each $a \in \Sigma$, every node in T has at most one outgoing edge whose label starts with a.*

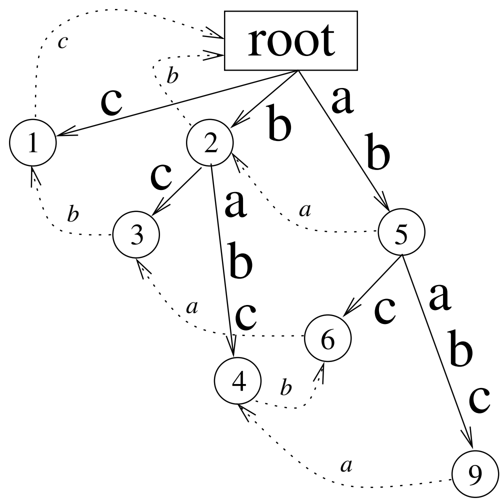


Definition 3 (path(n)). *If n is a node in Σ^+-tree T, then path(n) is the string built by concatenating all edge labels from the root to n. It is a unique identifier for the tree position.*

Definition 4 ($words(T)$). *A string u is in $words(T)$, if there is a node n in T s.t. $\exists v \in \Sigma^*. uv = \mathsf{path}(n)$.*

Suffix Trees and Suffix Links

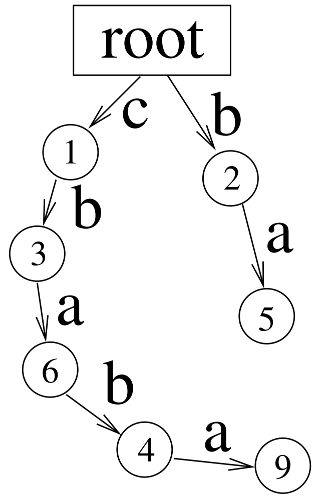
Definition 5 (Suffix tree). *A suffix tree of string t is a Σ^+-tree with $words(T) = \{u | u$ is a substring of $t\}$.*



Definition 6 (Suffix Link). *A suffix link is an auxiliary edge from node n to node m where m is the node s.t.* $\mathsf{path}(m)$ *is the longest proper suffix of* $\mathsf{path}(n)$ *represented by a node in T.*

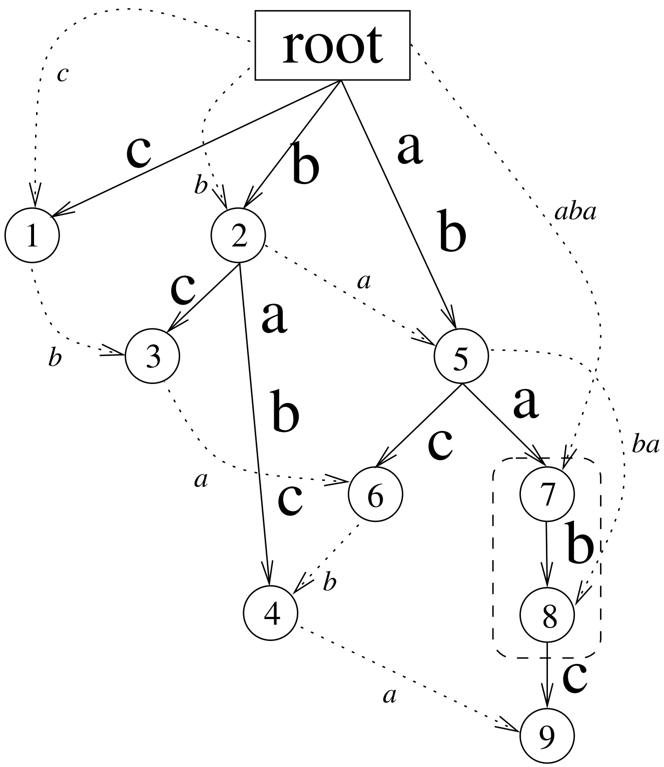
Reverse Tree and Affix Trees

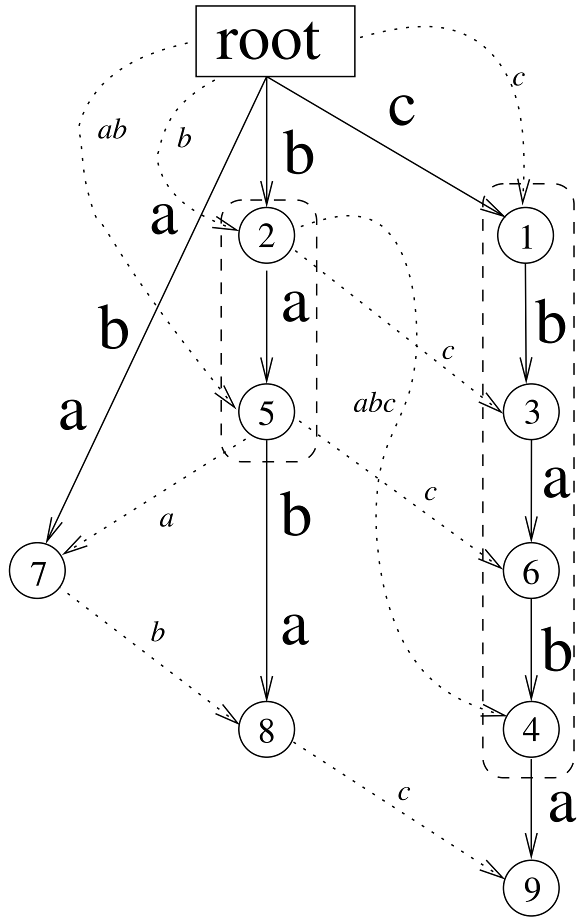
Definition 7 (Reverse tree T^{-1}). *The reverse tree T^{-1} of a Σ^+-tree T augmented with suffix links is defined as the tree that is formed by the suffix links of T, where the direction of each link is reversed, but the label is kept.*



Definition 8 (Affix tree). *An affix tree T of a string t is a Σ^+-tree s.t.*
$words(T) = \{u|u$ *is a substring of* $t\}$ *and*
$words(T^{-1}) = \{u|u$ *is a substring of* $t^{-1}\}$.

Affix Trees





Previous Work

- Weiner, McCreight: linear suffix tree construction
- Ukkonen: linear on-line suffix tree construction, reference pairs, open edges
- Giegerich and Kurtz: relationship between suffix tree and its reverse tree through suffix links
- Stoye: affix tree data structure
- Blumer et al.: DAWG, c-DAWG with suffix links invariant under reversal

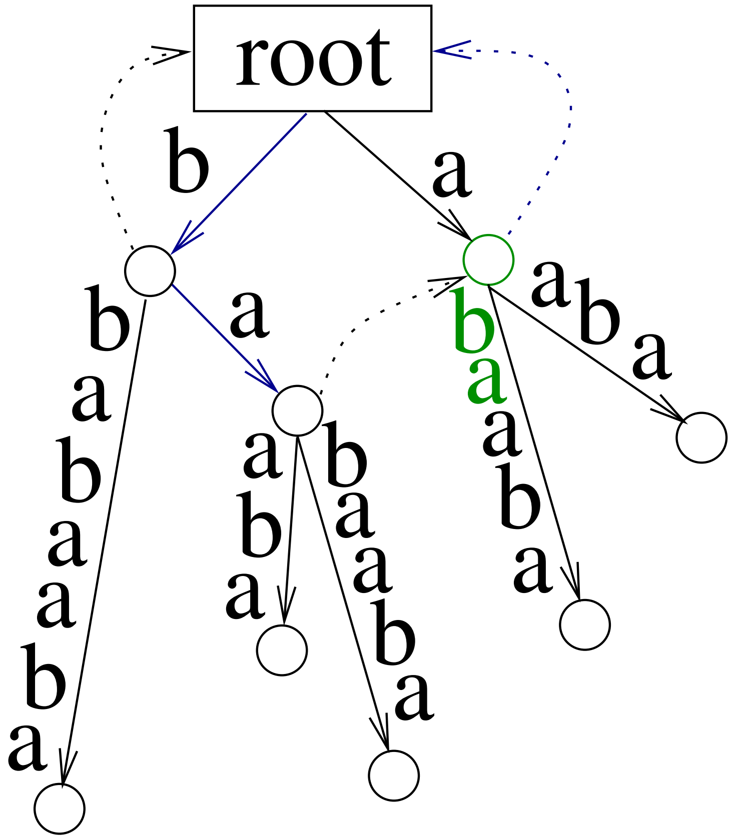
1. Introduction

2. $\boxed{\textbf{Construction of Suffix Trees}}$

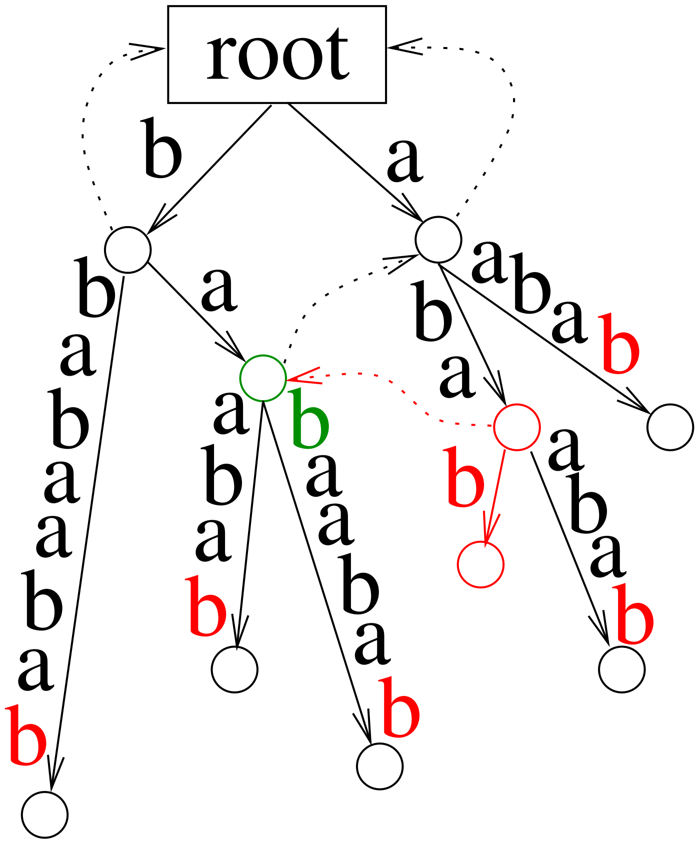
3. Construction Affix Trees

4. Complexity

On-Line Suffix Tree Construction

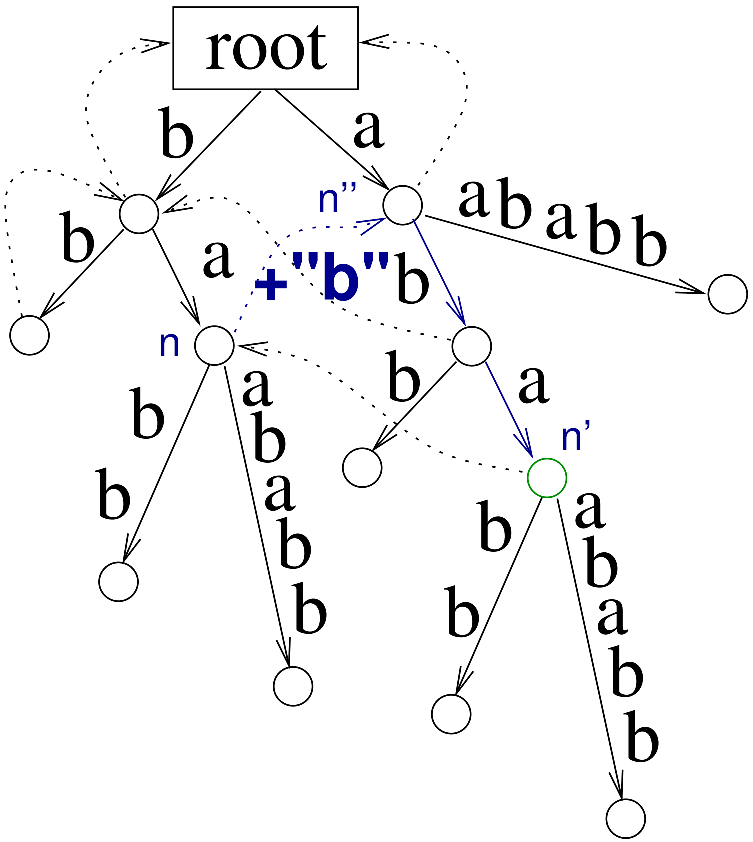


CST$(\mathrm{bbabaaba})$

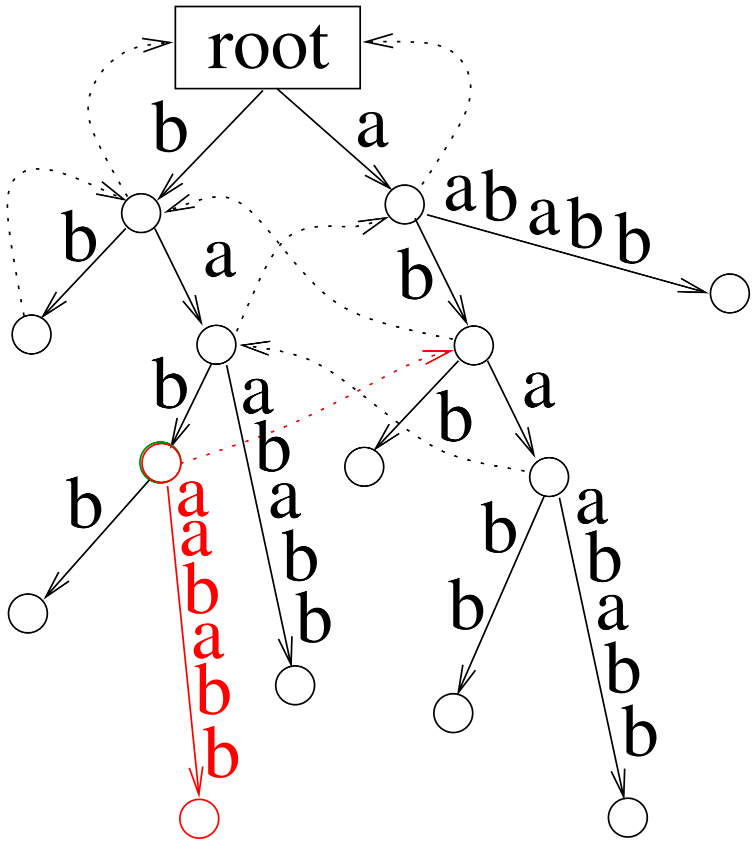


CST$(\mathrm{bbabaabab})$

Anti-On-Line Suffix Tree Construction



$\mathsf{CST}(\mathrm{abaababb})$

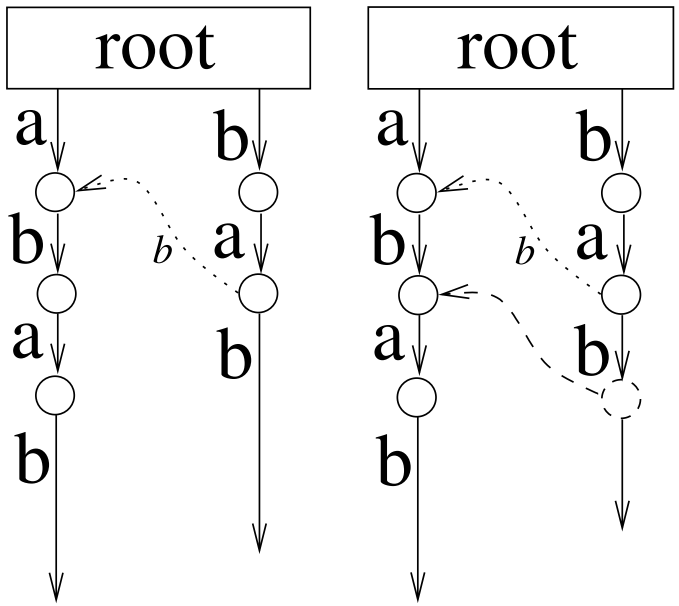


$\mathsf{CST}(\mathrm{babaababb})$

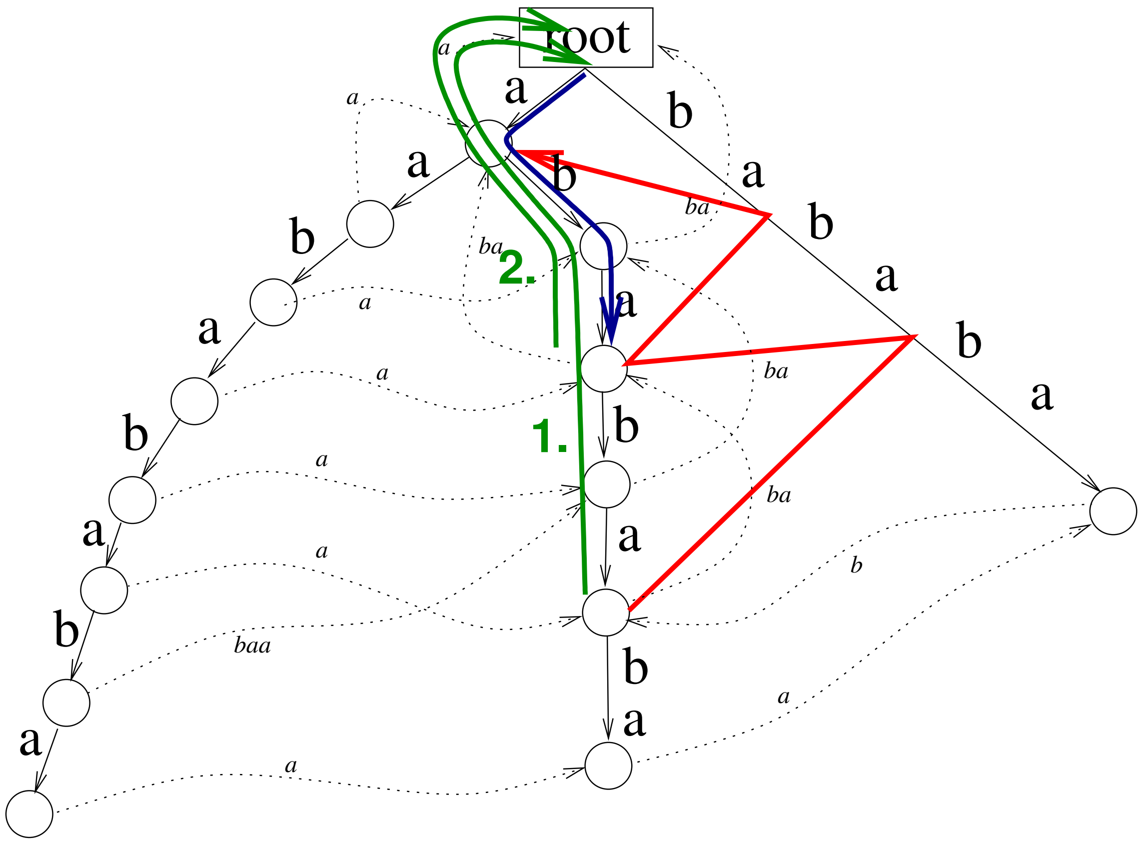
Complexity of Suffix Tree Construction

Lemma 1. *Ukkonen's algorithm constructs CST(t) on-line in time $\mathcal{O}(|t|)$.*

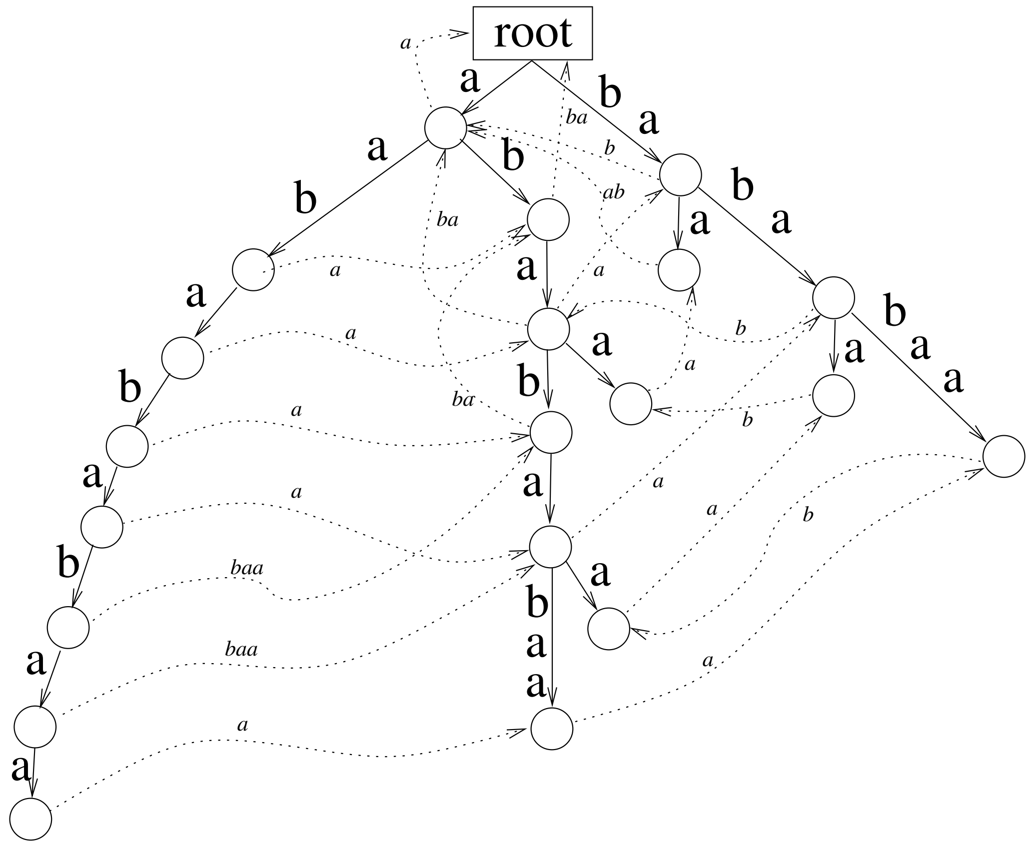
Lemma 2. *With the additional information of knowing the length of the active prefix for any suffix s of t before inserting it, it takes $\mathcal{O}(|t|)$ time to construct CST(t) in an anti-on-line manner.*



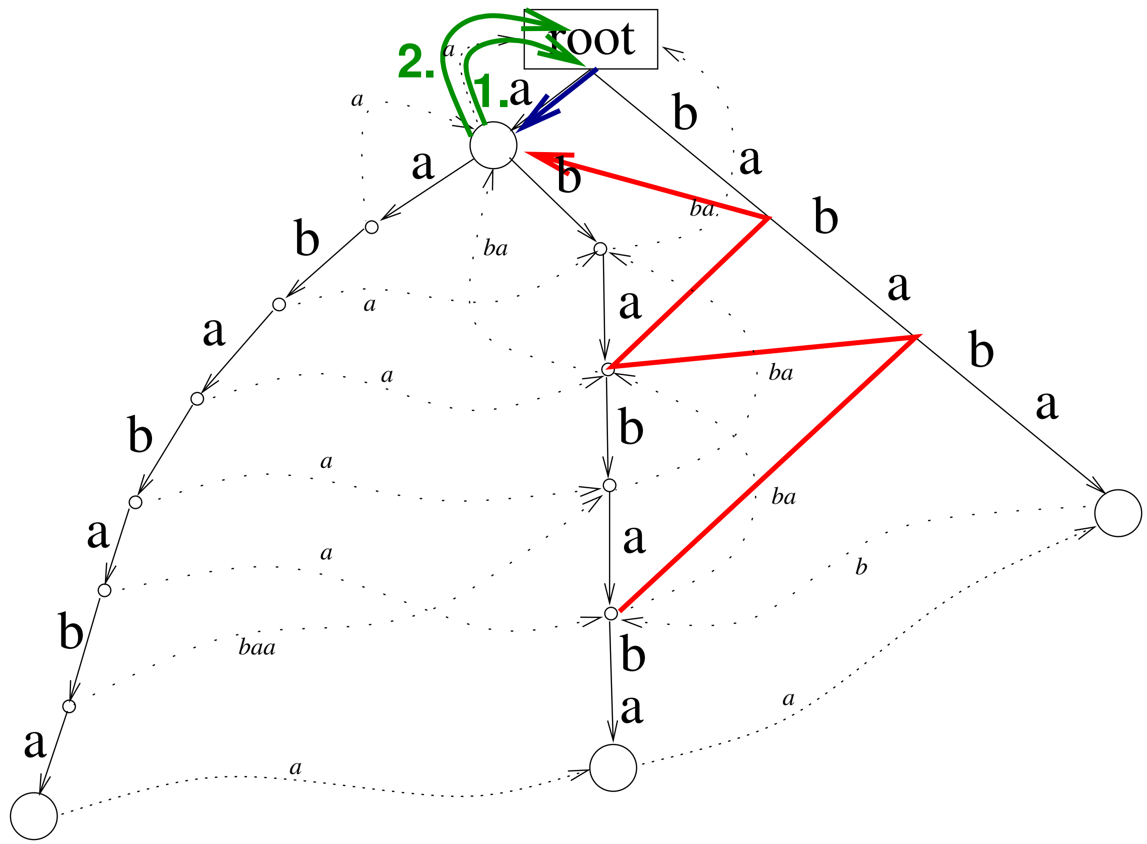
The Problem in Constructing Affix Trees



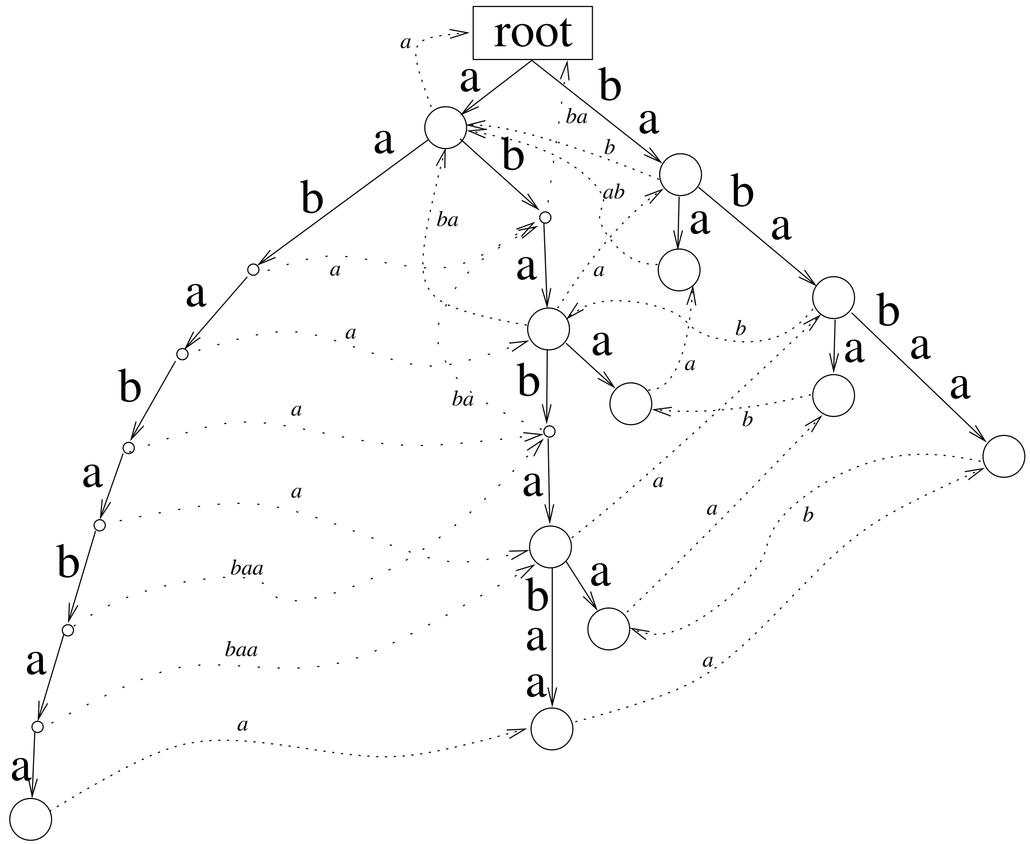
The Problem in Constructing Affix Trees (continued)



The Solution: Paths



The Solution: Paths (continued)



Additional Steps in Affix Tree Construction

- Updating Paths:

  Lemma 3. *The prefix parent of the active suffix leaf is (also) a prefix node.*

- Keeping track of the active suffix point, the active prefix point, the active suffix leaf, and the active prefix leaf:

  Lemma 4. *The active prefix will grow in the iteration from t to ta, iff the new active suffix of ta is represented by a prefix leaf in CAT(t).*

- Deleting Nodes

Summary of all Steps

1. Remove the suffix link from the active suffix link to s.

2. Lengthen the text, thereby lengthening all open edges.

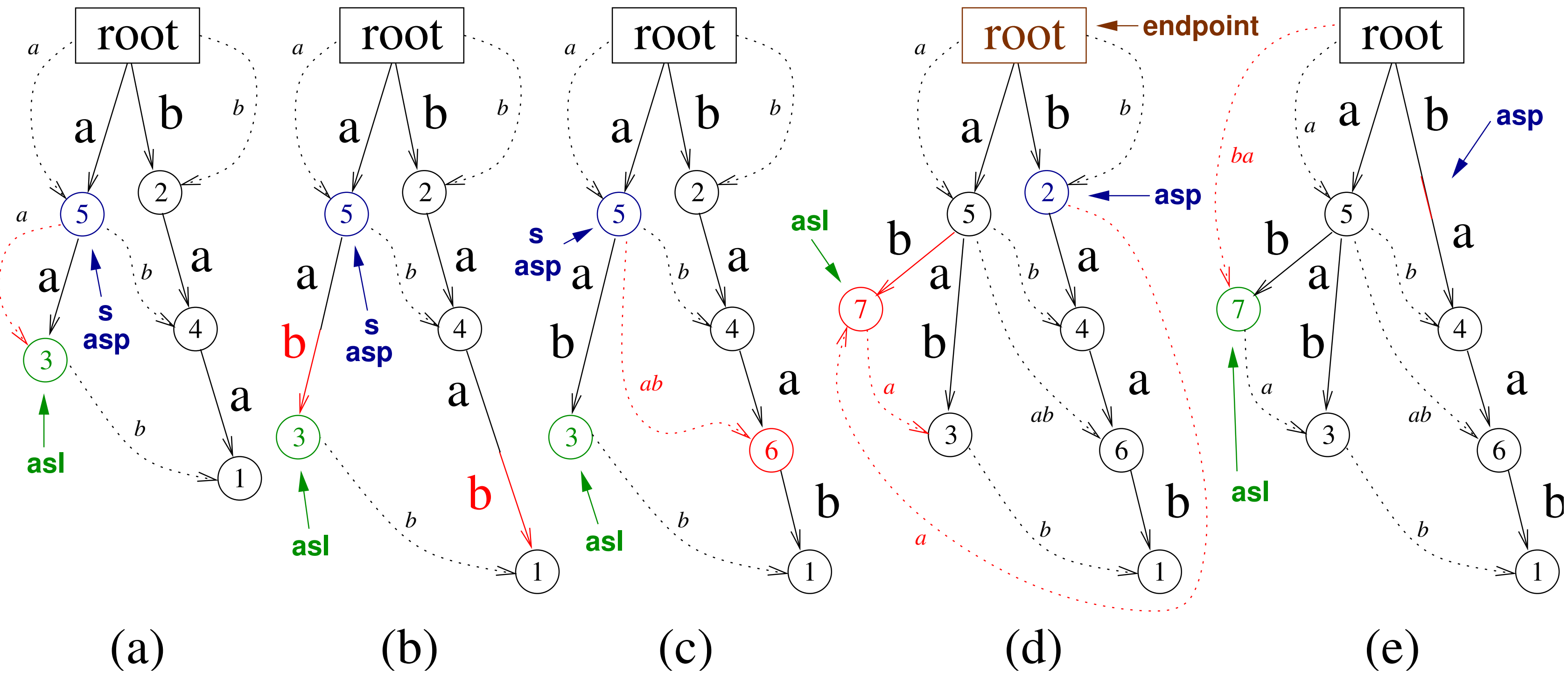
3. Insert the prefix node for t as suffix parent of $\overline{ta}$ and link it to s.

4. Insert relevant suffixes and update suffix links.

5. Make the location of the new active suffix $\alpha(ta)$ explicit and add a suffix link from the new active suffix link to it.

6. Update the active prefix, possibly deleting a node.

Example of a Single Iteration



1. Introduction

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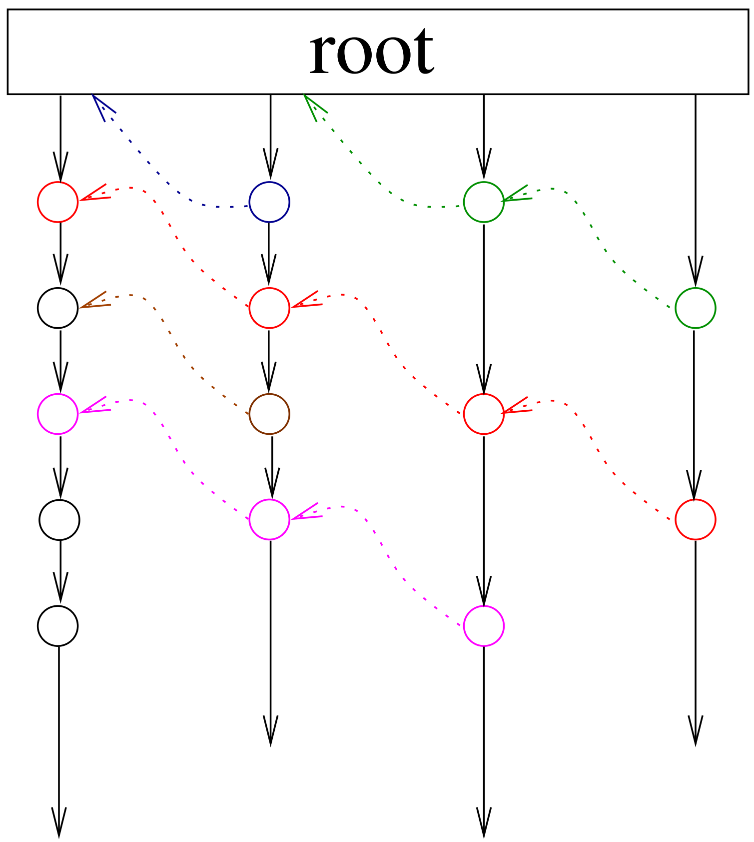
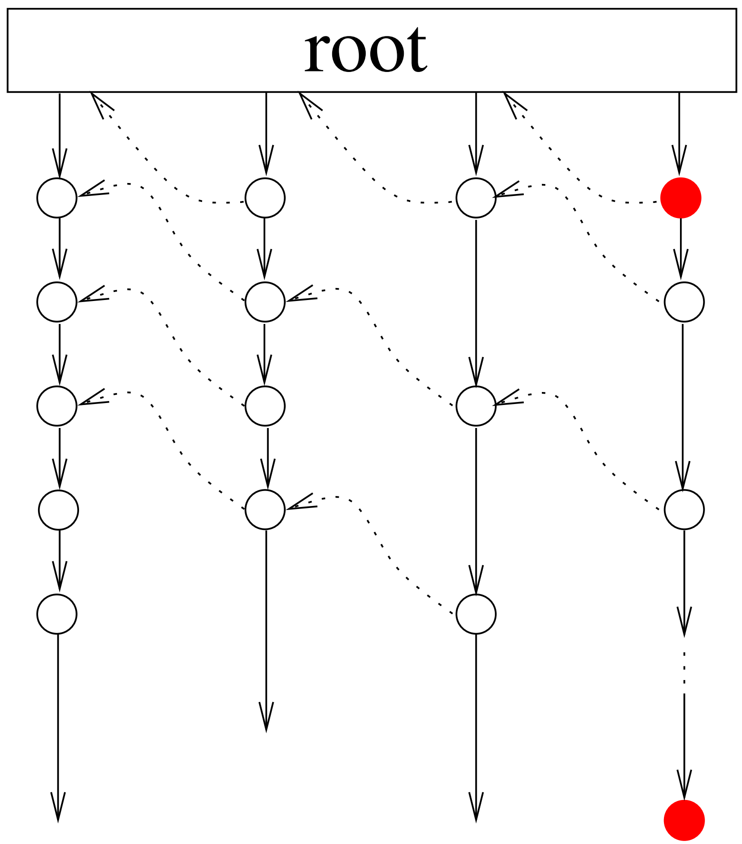
Complexity of Affix Tree Construction

Theorem 1. *CAT*(t) *can be constructed in an on-line manner from left to right or from right to left in time* $\mathcal{O}(|t|)$.

Theorem 2. *Bidirectional construction of affix trees has linear time complexity.*

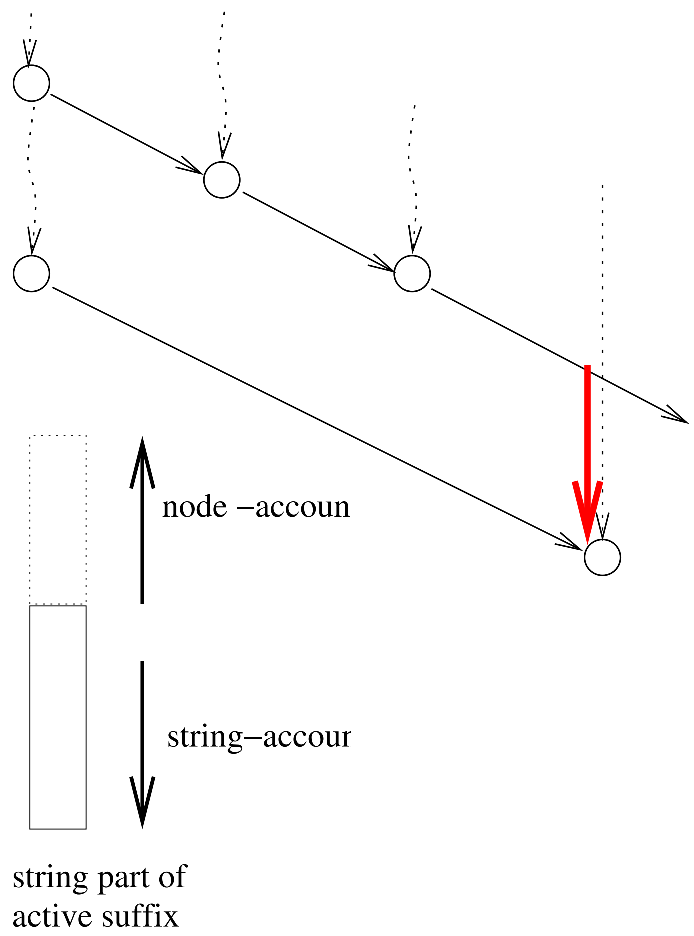
Bidirectional Construction - Changes to the Active Prefix Point





Bidirectional Construction - Changes to the Active Suffix Point

- Growth of the active suffix in a reverse iteration adds node-accounted part.
- Insertion of suffix nodes in a reverse iteration is only relevant to node-accounted part.
- node-accounted part can not be nested.



Conclusion

- Affix trees are a natural extension of suffix trees.
- Construction can be done in linear time, on-line and bidirectional.
- Affix tree augmented by paths behave like suffix trees.
- The view can be switched from the suffix to the prefix tree at any time.
- Right branching and left branching substrings represented in one structure.